## HP 35s Surveying Programs

HP 35s
Scientific Calculator

| FN= | ISG | RTN | $x ? y$ |  | FLAGS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R/S | GTO | X $=0$ | MODE | DISPLAY |  |
| PRGM A | DSE B | LBL C | $x \geqslant 0$ D |  |  |
| $\boldsymbol{x}$ § | VIEW | INPUT | ARG | $<$ | $>$ |
| RCL | R! | $x<1+y$ | i |  |  |
| STO | R1 E | PSE F | $\theta$ G |  | $\checkmark$ |
| HYP | $\pi$ | INTG | $x \sqrt{y}$ | IOG | $10^{x}$ |
| SIN | COS | TAN | $\sqrt{x}$ | $y^{x}$ | 1/x |
| ASIN H | ACOS 1 | ATAN 1 | $x^{2} \mathrm{~K}$ | IN 1 | $e^{x} \mathrm{M}$ |
| SHOW |  | $=$ | -ENG | G ENG $\rightarrow$ | $\rightarrow$ UNDO |
| ENTER |  | +/- | E | () | $\leftarrow$ |
| LASTX |  | ABS N | RND 0 | [1 | P CIEAR |
| $f$ | $\sim{ }^{\circ} \mathrm{F}$ | HMS $\rightarrow$ |  | $\rightarrow$ RAD | \%CHG |
| $\begin{aligned} & \text { EQN } \\ & \text { SOIVE } 0 \end{aligned}$ | 7 |  |  | 9 | $\div$ |
|  | $\rightarrow{ }^{\circ} \mathrm{C}$ | $\rightarrow \mathrm{H}$ | 55 | $\rightarrow$ DEG I | \% |
|  | $\rightarrow \mathrm{lb}$ |  |  | $\rightarrow$ in | nCr |
| $\leftarrow$ | 4 |  |  | 6 | $\times$ |
|  | $\rightarrow \mathrm{kg}$ U | $\rightarrow$ KN | v | $\rightarrow \mathrm{cm}$ W | nPr |
|  | IOGIC |  |  | SEED | L.R |
| $\rightarrow$ | 1 | 2 |  | 3 | - |
|  | BASE X |  | Y | RAND z | SUMS |
| OFF | , | /c |  | $\Sigma-$ | $\bar{x}, \bar{y}$ |
| C | 0 |  |  | $\Sigma+$ | + |
| ON | SPACE (1) | FDIS | (1) | $\underline{1}$ | s, $\sigma$ |

CLOSURE with Accuracy, Area and double-missing distance
Coordinate RADIATIONS with rotation and scale Coordinate JOINS
Radiations from OFFSETS
RESECTION
ADJUSTMENT - Bowditch and Crandall

## HP35s SURVEYING PROGRAMS

1. The following programs have been collated for the use of students in the Surveying and Geospatial Science programs in the School of Mathematical and Geospatial Sciences, RMIT University. As always, it is the user's responsibility to ensure that the programs are installed correctly and then checked. Also, do not alter programs unless you are aware of what LABELS are being used or whether GTO and BRANCHING label addresses will be affected; because by doing so you may dramatically affect the way they work and hence obtain incorrect answers.
2. The following two programs under LABEL $Z$ are critical and must be kept in your HP35s at all times. Do not delete them!

- RECTANGULAR $\rightarrow$ POLAR XEQ Z002
- POLAR $\rightarrow$ RECTANGULAR XEQ Z015

These programs are software replacements for the Polar $\leftrightarrows$ Rectangular conversion functions that were present on the HP33s and HP32s calculators and have not been implemented on the HP35s.
3. The following are a 'suite' of surveying computation programs that will be useful in the field and office. Some (Closure, Radiations, Joins, Offsets) have a heritage extending back to HP desktop-computer programs from the 1970's written by Bodo Taube of Francis O'Halloran, Surveyors. And Bodo Taube's programs were (and are) models of efficiency. Others are more recent.
Each program has a set of User Instructions, with examples and relevant formula and HP35s Program Sheets listing the program steps (that you may key into your calculator), storage registers used and program notes.

- CLOSURE XEQ C001
- RADIATIONS XEQR001
- JOINS XEQ J001
- OFFSETS XEQ 0001
- RESECTION XEQ S001
- ADJUSTMENT XEQ A001

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## HP35s POLAR $\leftrightarrows R E C T A N G U L A R ~ C O N V E R S I O N S ~$

The following programming code is a software replacement for the POLAR $\leftrightarrows R E C T A N G U L A R$ conversion functions that were present on the HP33s and HP32s calculators and have not been implemented on the HP35s.

This code was made available through The Museum of HP Calculators and appeared in HP Forum Archive 17 (22-Aug-2007)
http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv017.cgi?read=122519

| RECTANGULAR $\rightarrow$ POLAR | XEQ Z002 | XEQ | 3 | 0 | 0 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| POLAR $\rightarrow$ RECTANGULAR | XEQ Z015 | XEQ | 3 | 0 | 1 | 5 |




What do these pieces of code do?

## RECTANGULAR $\rightarrow$ POLAR



## POLAR $\rightarrow$ RECTANGULAR



The contents of registers $Z$ and $T$ remain unchanged for both conversions.

## MISSING BEARING \& DISTANCE OR DOUBLE MISSING DISTANCE (BEARING INTERSECTION) WITH AREA

## PRESS XEQ C001 TO RUN PROGRAM

Notes: 1. For missing bearing and distance, the missing line must be the last line in the closure.
2. For double missing distance, the missing distances must be on the last two lines of the closure.
3. Missing elements must be input as zero, i.e., if the bearing is unknown then enter 0 when requested and if the distance is unknown enter 0 when requested.
4. Bearings of lines that are $0^{\circ} 00^{\prime} 00^{\prime \prime}$ must be entered as $360^{\circ} 00^{\prime} 00^{\prime \prime}$

$B_{n}^{1}-B_{n-1}^{n}= \pm(180-\gamma)$
$B_{1}^{n-1}-B_{n}^{1}= \pm(180-\alpha)$
$\sin (180-\gamma)=\sin \gamma$
$\sin (180-\alpha)=\sin \alpha$
$\pm a=\frac{c \sin \alpha}{\sin \gamma}$


AREA ALGORITHM $\Delta$ Area $_{k}=-\frac{1}{2}\left\{\Delta N_{k} \sum_{i=1}^{k} \Delta E_{i}-\Delta E_{k} \sum_{i=1}^{k} \Delta N_{i}\right\}$

## EXAMPLES

1. Closure with: (i) misclose bearing and distance;
(ii) misclose east and north;
(iii) misclose accuracy; and
(iv) area


Figure $A B C D E F$ is section of road 20 m wide that is being excised from an allotment of land.
Check that the dimensions are correct and determine the area.
Starting with the line $A B$ and going clockwise around the figure, enter the bearing and distance of each side, remembering that the bearing of the last side $F A$ should be entered as $360^{\circ} 00^{\prime}$.

Enter 0 for the last bearing and 0 for the last side (you don't have to key anything in; just press $\mathrm{R} / \mathrm{S}$ at the prompts) since the last side (the misclose) is unknown.

The calculator will display: B = 136.0924 ( $136^{\circ} 09^{\prime} 24$ ") (the misclose bearing);
Press R/S
The calculator will display: $\mathrm{D}=0.0021$ (the misclose distance);
Press R/S
The calculator will display: 0.0014 (east misclose) 001
-0.0015 (north misclose); often shown as 002

## Press R/S

The calculator will display: $\quad \mathrm{R}=502,288.7039$ (this is the misclose accuracy ratio 1:502289)

## Press R/S

The calculator will display: $A=-9,926.0706$ (this is the area $9926 \mathrm{~m}^{2}$ )
(the negative sign is due to entering the figure clockwise)

## Press R/S

The calculator will display: B?
0.0000

Ready for the next closure.

## EXAMPLES

2. Closure with: (i) double missing and distance; and
(ii) area


Figure $A B C D E F$ is section of road 20 m wide that is being excised from an allotment of land.
Compute the missing distances $C D$ and $D E$, and the area.
Starting with the line EF and going clockwise around the figure, enter the bearing and distance of each 'known' side, remembering that the bearing of the side FA should be entered as $360^{\circ} 00^{\prime}$.

Enter the bearing of the side $C D$ and 0 for the distance (the 1st missing distance; you don't have to key anything in; just press R/S at the prompt).
Enter the bearing of the side $E D$. The calculator will now solve for the two missing distances $C D$ and $D E$.

The calculator will display: $\quad \mathrm{D}=20.0907$ (the 1st missing distance);
Press R/S
The calculator will display: $\quad \mathrm{D}=204.5581$ (the 2nd missing distance);
Press R/S
The calculator will display: $\quad A=-9,926.6036$ (this is the area $9926 \mathrm{~m}^{2}$ )
(the negative sign is due to entering the figure clockwise)
Press R/S
The calculator will display: B ?
0.0000

Ready for the next closure.

NOTE: For double missing distance closures, the missing sides must be the last two sides. To achieve this, some figures may need re-casting. In such cases, the areas of re-cast figures may not be correct. See the following example

## EXAMPLES

3. Closure with: (i) double missing and distance; and
(ii) area


Figure $A B C D E F$ is section of road 20 m wide that is being excised from an allotment of land.
Compute the missing distances $A B$ and $C D$, and the area.
Re-cast the figure so that the last two sides contain the missing distances


Starting with the line $D E$ and going clockwise around the re-cast figure, enter the bearing and distance of each 'known' side, remembering that the bearing of the side FA should be entered as $360^{\circ} 00^{\prime}$.

Enter the bearing of the side $B^{\prime} C$ and 0 for the distance (the 1st missing distance; you don't have to key anything in; just press R/S at the prompt).
Enter the bearing of the side $C D$. The calculator will now solve for the two missing distances $B^{\prime} C$ and $C D$.

The calculator will display: $\mathrm{D}=292.7520$ (the 1st missing distance);
Press R/S
The calculator will display: $\quad \mathrm{D}=20.0916$ (the 2nd missing distance);
Press R/S
The calculator will display: $\quad \mathrm{A}=18,126.6222$ (this is complete rubbish since the lines in the re-cast figure cross)

## AREA ALGORITHM

The algorithm for computing the area of a polygon can be derived by considering Figure A1, where the area is the sum of the trapeziums $b B C c, c C D d$ and $d D E e$ less the triangles $b B A$ and $A E e$.

The area can be expressed as

$$
\begin{aligned}
2 A= & {\left[\left(x_{2}-x_{1}\right)+\left(x_{3}-x_{1}\right)\right]\left[\left(y_{2}-y_{3}\right)\right] } \\
& +\left[\left(x_{3}-x_{1}\right)+\left(x_{4}-x_{1}\right)\right]\left[\left(y_{3}-y_{4}\right)\right] \\
& +\left[\left(x_{4}-x_{1}\right)+\left(x_{5}-x_{1}\right)\right]\left[\left(y_{4}-y_{5}\right)\right] \\
& -\left(x_{2}-x_{1}\right)\left(y_{2}-y_{1}\right) \\
& -\left(x_{5}-x_{1}\right)\left(y_{1}-y_{5}\right)
\end{aligned}
$$

Expanding (A1) then cancelling and rearranging terms gives

$$
\begin{aligned}
2 A & =x_{1}\left(y_{5}-y_{2}\right) \\
& +x_{2}\left(y_{1}-y_{3}\right) \\
& +x_{3}\left(y_{2}-y_{4}\right) \\
& +x_{4}\left(y_{3}-y_{5}\right) \\
& +x_{5}\left(y_{4}-y_{1}\right)
\end{aligned}
$$



Figure A1
which can be expressed as $2 A=\sum_{k=1}^{n}\left\{x_{k}\left(y_{k-1}-y_{k+1}\right)\right\}$

In Figure A2, the coordinate origin is shifted to $A$ where $x_{1}^{\prime}=y_{1}^{\prime}=0$ and the area, using (A2), is
$2 A=y_{2}^{\prime} x_{3}^{\prime}+y_{3}^{\prime} x_{4}^{\prime}-y_{3}^{\prime} x_{2}^{\prime}+y_{4}^{\prime} x_{5}^{\prime}-y_{4}^{\prime} x_{3}^{\prime}-y_{5}^{\prime} x_{4}^{\prime}$
Considering each side of the polygon to have components $\Delta x_{k}, \Delta y_{k}$ for $k=1$ to 5 , (A3) can be written as

$$
\begin{aligned}
2 A & =\Delta y_{1}\left(\Delta x_{1}+\Delta x_{2}\right) \\
& +\left(\Delta y_{1}+\Delta y_{2}\right)\left(\Delta x_{1}+\Delta x_{2}+\Delta x_{3}\right) \\
& -\left(\Delta y_{1}+\Delta y_{2}\right)\left(\Delta x_{1}\right) \\
& +\left(\Delta y_{1}+\Delta y_{2}+\Delta y_{3}\right)\left(\Delta x_{1}+\Delta x_{2}+\Delta x_{3}+\Delta x_{4}\right) \\
& -\left(\Delta y_{1}+\Delta y_{2}+\Delta y_{3}\right)\left(\Delta x_{1}+\Delta x_{2}\right) \\
& -\left(\Delta y_{1}+\Delta y_{2}+\Delta y_{3}+\Delta y_{4}\right)\left(\Delta x_{1}+\Delta x_{2}+\Delta x_{3}\right)
\end{aligned}
$$



Figure A2

Expanding and gathering terms gives

$$
\begin{array}{rll}
2 A= & \Delta y_{1}\left(3 \Delta x_{1}+3 \Delta x_{2}+2 \Delta x_{3}+\Delta x_{4}\right) & -\Delta y_{1}\left(3 \Delta x_{1}+2 \Delta x_{2}+\Delta x_{3}\right) \\
& +\Delta y_{2}\left(2 \Delta x_{1}+2 \Delta x_{2}+2 \Delta x_{3}+\Delta x_{4}\right) & -\Delta y_{2}\left(3 \Delta x_{1}+2 \Delta x_{2}+\Delta x_{3}\right) \\
& +\Delta y_{3}\left(\Delta x_{1}+\Delta x_{2}+\Delta x_{3}+\Delta x_{4}\right) & -\Delta y_{3}\left(2 \Delta x_{1}+2 \Delta x_{2}+\Delta x_{3}\right) \\
& -\Delta y_{4}\left(\Delta x_{1}+\Delta x_{2}+\Delta x_{3}\right)
\end{array}
$$

and cancelling terms and re-ordering gives

$$
\begin{align*}
2 A= & \Delta y_{1}\left(0+\Delta x_{2}+\Delta x_{3}+\Delta x_{4}\right) \\
& +\Delta y_{2}\left(-\Delta x_{1}+0+\Delta x_{3}+\Delta x_{4}\right) \\
& +\Delta y_{3}\left(-\Delta x_{1}-\Delta x_{2}+0+\Delta x_{4}\right)  \tag{A4}\\
& +\Delta y_{4}\left(-\Delta x_{1}-\Delta x_{2}-\Delta x_{3}+0\right)
\end{align*}
$$

This equation for the area can also be expressed as a matrix equation

$$
2 A=\left[\begin{array}{llll}
\Delta y_{1} & \Delta y_{2} & \Delta y_{3} & \Delta y_{4}
\end{array}\right]\left[\begin{array}{cccc}
0 & 1 & 1 & 1  \tag{A5}\\
-1 & 0 & 1 & 1 \\
-1 & -1 & 0 & 1 \\
-1 & -1 & -1 & 0
\end{array}\right]\left[\begin{array}{l}
\Delta x_{1} \\
\Delta x_{2} \\
\Delta x_{3} \\
\Delta x_{4}
\end{array}\right]
$$

By studying the form of equations (A4) and (A5), the following algorithm for calculating the $k=n-1$ area components $A_{k}$ for a polygon of $n$ sides may be deduced as

$$
\begin{equation*}
A_{k}=\frac{1}{2}\left\{\Delta x_{k} \sum_{i=1}^{k} \Delta y_{i}-\Delta y_{k} \sum_{i=1}^{k} \Delta x_{i}\right\} \text { where } k=1,2,3, \ldots n-1 \tag{A6}
\end{equation*}
$$

Equation (A6) is an efficient way to accumulate the area of a polygon given the coordinate components of the sides. By studying the algorithm, it can be seen that $A_{1}=A_{n}=0$ and hence the area of a polygon is accumulated without having to deal with the last side. This makes it a very useful area algorithm for simple closure programs where the last side is often the missing side in the polygon. In addition, it can be seen that each area component $A_{k}$ is a triangle with one vertex at the starting point and the line $k$, with components $\Delta x_{k}, \Delta y_{k}$, the opposite side.

Rearranging equation (A6) and expressing the components of lines as $\Delta E$ and $\Delta N$ where $E$ and $N$ are east and north respectively gives the area algorithm used in the HP35s Closure Program

$$
\begin{equation*}
A_{k}=-\frac{1}{2}\left\{\Delta N_{k} \sum_{i=1}^{k} \Delta E_{i}-\Delta E_{k} \sum_{i=1}^{k} \Delta N_{i}\right\} \text { where } k=1,2,3, \ldots n-1 \tag{A7}
\end{equation*}
$$

| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C001 | LBL C |  |  |  |  |
| C002 | CLVARS |  | START NEW | CLOSURE |  |
| C003 | CLE |  |  |  |  |
| C004 | 0 |  | NEW LINE O | F CLOSURE |  |
| C005 | STO B |  |  |  |  |
| C006 | STO D |  |  |  |  |
| C007 | INPUT B | Enter Bea | ing |  |  |
| C008 | HMS $\rightarrow$ |  |  |  |  |
| C009 | STO B |  |  |  |  |
| C010 | STO C |  |  |  |  |
| C011 | INPUT D | Enter Dis | ance |  |  |
| C012 | ST0+R | accumulat | distances |  |  |
| C013 | RCL B | Brg | Dist |  |  |
| C014 | + | Brg+Dist |  |  |  |
| C015 | $x=0$ ? | test to s | $e$ if both | Brg \& Dist | zero |
| C016 | GT0 C068 | go for mi | sing beari | ng \& dista |  |
| C017 | RCL D |  |  |  |  |
| C018 | $x=0$ ? | test to s | $e$ if Dist | is zero |  |
| C019 | GTO C042 | go for do | ble missing | g distance |  |
| C020 | XEQ C022 | compute a | ea contrib | ution for |  |
| C021 | GTO C004 | go for ne | $t$ line of | closure |  |
| C022 | RCL B | Brg | AREA SUBRO | UTINE |  |
| C023 | RCL D | Dist | Brg |  |  |
| C024 | XEQ Z015 | $\Delta \mathrm{N}$ | $\Delta \mathrm{E}$ |  |  |
| C025 | $\Sigma+$ | n | $\Delta \mathrm{E}$ |  |  |
| C026 | R $\downarrow$ | $\Delta \mathrm{E}$ |  |  |  |
| C027 | LASTX | $\Delta \mathrm{N}$ | $\Delta \mathrm{E}$ |  |  |
| C028 | $\Sigma y$ | $\Sigma(\Delta \mathrm{E})$ | $\Delta N$ | $\Delta \mathrm{E}$ |  |
| C029 | $\times$ | $\Delta N(\Sigma(\Delta E))$ | $\Delta \mathrm{E}$ |  |  |
| C030 | $x<>y$ | $\Delta \mathrm{E}$ | $\Delta N(\Sigma(\Delta E))$ |  |  |
| C031 | $\Sigma x$ | $\Sigma(\Delta N)$ | $\Delta \mathrm{E}$ | $\Delta N(\Sigma(\Delta E))$ |  |
| C032 | $\times$ | $\Delta E(\Sigma(\Delta N))$ | $\Delta N(\Sigma(\Delta E))$ |  |  |
| C033 | - | $\Delta N(\Sigma(\Delta E))$ | $\Delta E(\Sigma(\Delta N))$ |  |  |
| C034 | 2 |  |  |  |  |
| C035 | $\div$ | area comp | nent |  |  |
| C036 | ST0+A | accumulat | area |  |  |
| C037 | RTN |  |  |  |  |
| C038 | $\Sigma y$ | $\Sigma(\Delta \mathrm{E})$ | BRG \& DIST | SUBROUTIN |  |
| C039 | $\Sigma x$ | $\Sigma(\Delta N)$ | $\Sigma(\Delta \mathrm{E})$ |  |  |
| C040 | XEQ Z002 | Dist | Brg |  |  |
| C041 | RTN |  |  |  |  |


| LINE | STEP | X | $Y$ | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C042 | 0 |  | DOUBLE MISSING DISTANCE |  |  |
| C043 | STO B |  |  |  |  |
| C044 | INPUT B | Enter 2nd Bearing $B_{n}^{1}$ |  |  |  |
| C045 | HMS $\rightarrow$ |  |  |  |  |
| C046 | STO B | $B_{n}^{1}$ |  |  |  |
| C047 | RCL C | $B_{n-1}^{n}$ | $B_{n}^{1}$ |  |  |
| C048 | - | $\pm(180-\gamma)$ |  |  |  |
| C049 | SIN | $\pm \sin \gamma$ |  |  |  |
| C050 | XEQ C038 | c | $B_{1}^{n-1}$ | $\pm \sin \gamma$ |  |
| C051 | $x<>y$ | $B_{1}^{n-1}$ | c | $\pm \sin \gamma$ |  |
| C052 | RCL B | $B_{n}^{1}$ | $B_{1}^{n-1}$ | c | $\pm \sin \gamma$ |
| C053 | - | $\pm(180-\alpha)$ | c | $\pm \sin \gamma$ |  |
| C054 | SIN | $\pm \sin \alpha$ | c | $\pm \sin \gamma$ |  |
| C055 | $\times$ | $\pm c \sin \alpha$ | $\pm \sin \gamma$ |  |  |
| C056 | $x<>y$ | $\pm \sin \gamma$ | $\pm c \sin \alpha$ |  |  |
| C057 | $\div$ | $\pm a$ (1st missing distance) |  |  |  |
| C058 | STO D | $\pm a$ |  |  |  |
| C059 | RCL C | $B_{n-1}^{n}$ | $\pm a$ |  |  |
| C060 | STO B |  |  |  |  |
| C061 | XEQ C022 | compute area contribution for line |  |  |  |
| C062 | VIEW D | 1st Missing Distance |  |  |  |
| C063 | XEQ C038 |  |  |  |  |
| C064 | STO D |  |  |  |  |
| C065 | VIEW D | 2nd Missing Distance |  |  |  |
| C066 | VIEW A | Area |  |  |  |
| C067 | GT0 C002 |  |  |  |  |
| C068 | XEQ C038 | Dist | Brg | MISSING | G \& DIST |
| C069 | STO D |  |  |  |  |
| C070 | ST0 $\div$ R |  |  |  |  |
| C071 | $x<>y$ |  |  |  |  |
| C072 | 180 |  |  |  |  |
| C073 | + |  |  |  |  |
| C074 | $\rightarrow$ HMS | Brg |  |  |  |
| C075 | STO B |  |  |  |  |
| C076 | VIEW B | Missing Bearing |  |  |  |
| C077 | VIEW D | Missing Distance |  |  |  |
| C078 | $\Sigma y$ | $\Sigma(\Delta \mathrm{E})$ |  |  |  |
| C079 | +/- | $-\Sigma(\Delta \mathrm{E})$ |  |  |  |
| C080 | $\Sigma x$ | $\Sigma(\Delta N)$ | $-\Sigma(\Delta \mathrm{E})$ |  |  |
| C081 | +/- | $-\Sigma(\Delta N)$ | $-\Sigma(\Delta \mathrm{E})$ |  |  |
| C082 | STOP | N miscl. | E miscl. |  |  |


| LINE | STEP | X | Y | Z | T |
| :---: | :--- | :--- | :--- | :--- | :---: |
| C083 | VIEW R | Misclose Accuracy $1: x$ |  |  |  |
| C084 | VIEW A | Area |  |  |  |
| C085 | GT0 C002 |  |  |  |  |

## STORAGE REGISTERS

| $\mathbf{A}$ | Area |
| :--- | :--- |
| $\mathbf{B}$ | Bearing |
| $\mathbf{C}$ | Bearing |
| $\mathbf{D}$ | Distance |
| $\mathbf{R}$ | Cumulative distance; closure accuracy |
|  |  |

## PROGRAM LENGTH AND CHECKSUM

## LN = 261; CK = D83C

$\star$ Length \& Checksum: $\square \square \square 2$; $\square$ ENTER (Hold)

## PROGRAM NOTES

| Lines co22 to co37 | is an area subroutine that also <br> accumulates the east and north <br> components of lines |
| :--- | :--- |
| Lines co38 to co41 |  |
| is a subroutine to calculate a bearing |  |
| and distance from east and north |  |

The calculator must contain LBL Z which contains the Polar to Rectangular routines

## USER INSTRUCTIONS COORDINATE RADIATIONS PROGRAM

1. To start program press XEQ R001

| 2. | Display | $\begin{aligned} & E ? \\ & 0.0000 \end{aligned}$ | Enter: | East coordinate of traverse point; then press | R/S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3. | Display | $\begin{aligned} & \mathrm{N} \text { ? } \\ & 0.0000 \end{aligned}$ | Enter: | North coordinate of traverse point; then press | R/S |
| 4. | Display | $\begin{aligned} & \text { R? } \\ & 0.0000 \end{aligned}$ | Enter: <br> [If no r | Rotation ( $\pm$ D.MMSS); then press R/S tation to be applied, press R/S and rotation $=$ |  |
| 5. | Display | $\begin{aligned} & \text { S? } \\ & 1.0000 \end{aligned}$ | Enter: <br> [If no s | Scale Factor; then press R/S <br> ale factor to be applied, press R/S and scale | $\text { factor }=1 \text { ] }$ |
| 6. | Display | $\begin{aligned} & \text { B? } \\ & 0.0000 \end{aligned}$ | Enter: | Radiation Bearing (D.MMSS); then press | R/S |
| 7. | Display | $\begin{aligned} & \mathrm{D} \text { ? } \\ & 0.0000 \end{aligned}$ | Enter: <br> [If next | Radiation Distance; then press R/S Instrument Point, enter distance with a negativ | ive sign.] |

7A If Rotation and Scale not $0^{\circ}$ and 1 ; new bearing and distance displayed at successive R/S
8. East and North coordinate displayed at successive R/S. GoTo step 6.

In the example traverse below, with rotation $=0^{\circ}$ and scale $=1$, start at $A$, compute the coordinates of $A 1$ and $A 2$; jump to $B$, compute coordinates of $B 1$ and $B 2$; then to $C$ and the coordinates of $C 1$, $C 2$ and $C 3$. The values in parentheses are for rotation $=+2^{\circ} 18^{\prime} 35$ " and scale factor $=1.002515$. (Distances and coordinates are rounded to nearest mm .)



| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R044 | VIEW B | Rotated Bearing (D.MMSS) |  |  |  |
| R045 | VIEW D | Scaled Distance |  |  |  |
| R046 | RCL B |  |  |  |  |
| R047 | HMS $\rightarrow$ | Brg |  |  |  |
| R048 | RCL D | Dist | Brg |  |  |
| R049 | XEQ Z015 | $\Delta \mathrm{N}$ | $\Delta \mathrm{E}$ |  |  |
| R050 | STO+N | $\Delta \mathrm{N}$ | $\Delta \mathrm{E}$ |  |  |
| R051 | FS? 1 | Test for new Instrument Point |  |  |  |
| R052 | $\Sigma+$ | n | $\Delta \mathrm{E}$ | Yes! n |  |
| R053 | $x<>y$ | $\Delta \mathrm{E}$ | n |  |  |
| R054 | ST0+E |  |  |  |  |
| R055 | VIEW E | East |  |  |  |
| R056 | VIEW N | North |  |  |  |
| R057 | $\Sigma x$ | North coord of Instrument Point |  |  |  |
| R058 | STO N |  |  |  |  |
| R059 | Ey | East coord of Instrument Point |  |  |  |
| R060 | STO E |  |  |  |  |
| R061 | GT0 R024 |  |  |  |  |

## STORAGE REGISTERS

| $\mathbf{B}$ | Bearing(D.MMSS); Bearing(Degree); Rotated Brg |
| :--- | :--- |
| $\mathbf{D}$ | Distance; Scaled Distance |
| $\mathbf{E}$ | East coordinate |
| $\mathbf{N}$ | North coordinate |
| $\mathbf{R}$ | Rotation (D.MMSS); Rotation (Degrees) |
| $\mathbf{S}$ | Scale factor |
| $\mathbf{T}$ | T=999 if Rotation <br> T $\neq 999$ if any other Rotation and Scale Factor |
| $\boldsymbol{\Sigma} \boldsymbol{x}$ | North coordinate of Instrument Point |
| $\boldsymbol{\Sigma} \boldsymbol{y}$ | East coordinate of Instrument Point |

## PROGRAM LENGTH AND CHECKSUM

$L N=191 ; C K=22 C 1$
$\star$ Length \& Checksum: $\leftarrow \square \boxed{\square} ; \quad \leftarrow$ ENTER (Hold)

## PROGRAM NOTES



The calculator must contain LBL Z which contains the Polar to Rectangular routines

XEQ Z015 on line R049 is the Polar $\rightarrow$ Rectangular conversion

## USER INSTRUCTIONS COORDINATE JOINS PROGRAM

1. To start program press XEQ J001
2. Display E? Enter: East coordinate of Instrument Point; then press R/S
3. Display N? Enter: North coordinate of Instrument Point; then press R/S
0.0000
4. Display E? Enter: East coordinate of next point; then press R/S
$\begin{array}{lll}\text { 5. Display } & \begin{array}{l}\text { N? } \\ \\ 0.0000\end{array} & \begin{array}{l}\text { Enter: North coordinate of next point; then press R/S } \\ \text { [If next Instrument Point, enter Northing with negative sign.] }\end{array}\end{array}$
5. Bearing (D.MMSS) and Distance displayed at successive R/S. GoTo step 4.

In the example traverse below, start at $A$, compute the radiations (bearings and distances) to $A 1$ and $A 2$; jump to $B$, compute radiations to $B 1$ and $B 2$; then to $C$ and the radiations to $C 1, C 2$ and $C 3$. (The computed bearings and distances are rounded to the nearest 5 mm and 10 " respectively.)


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| J001 | LBL J |  |  |  |  |
| J002 | CLVARS |  | START JOINS PROGRAM |  |  |
| J003 | INPUT E | Enter East coord of Instrument Point |  |  |  |
| J004 | INPUT N | Enter North coord of I.P. |  |  |  |
| J005 | RCL E | $\mathrm{E}_{\mathrm{i}}$ | NEW INSTRUMENT POINT |  |  |
| J006 | STO Y |  |  |  |  |
| J007 | RCL N | $\mathrm{N}_{\mathrm{i}}$ | $\mathrm{E}_{\mathrm{i}}$ |  |  |
| J008 | STO X |  |  |  |  |
| J009 | CF 1 |  | NEW POINT |  |  |
| J010 | 0 |  |  |  |  |
| J011 | STO E |  |  |  |  |
| J012 | STO N |  |  |  |  |
| J013 | INPUT E | Enter East coord of next point |  |  |  |
| J014 | INPUT N | Enter $\pm$ North coord of next point |  |  |  |
| J015 | $x<0$ ? | $\pm \mathrm{N}_{\mathrm{k}}$ | $\mathrm{E}_{\mathrm{k}}$ |  |  |
| J016 | SF 1 |  |  |  |  |
| J017 | ABS | $\mid N_{k}$ \| |  |  |  |
| J018 | STO N |  |  |  |  |
| J019 | RCL Y | $\mathrm{E}_{\mathrm{i}}$ |  |  |  |
| J020 | RCL E | $\mathrm{E}_{\mathrm{k}}$ | $\mathrm{E}_{\text {i }}$ |  |  |
| J021 | - | $\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{k}}$ |  |  |  |
| J022 | RCL X | $\mathrm{N}_{\mathrm{i}}$ | $\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{k}}$ |  |  |
| J023 | RCL N | $\mathrm{N}_{\mathrm{k}}$ | $\mathrm{N}_{\mathrm{i}}$ | $\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{k}}$ |  |
| J024 | XEQ Z002 | $\mathrm{N}_{\mathrm{i}}-\mathrm{N}_{\mathrm{k}}$ | $\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{k}}$ |  |  |
| J025 |  |  |  |  |  |
| J026 | STO D | Dist | Brg(k, i) |  |  |
| J027 | $x<>y$ | Brg(k,i) |  |  |  |
| J028 | 180 |  |  |  |  |
| J029 | + | Brg(i,k) |  |  |  |
| J030 | $\rightarrow$ HMS |  |  |  |  |
| J031 | STO B |  |  |  |  |
| J032 | VIEW B | Bearing |  |  |  |
| J033 | VIEW D | Distance |  |  |  |
| J034 | FS? 1 |  |  |  |  |
| J035 | GT0 J005 |  |  |  |  |
| J036 | GT0 J009 |  |  |  |  |

## STORAGE REGISTERS

| $\mathbf{B}$ | Bearing(D.MMSS) |
| :--- | :--- |
| $\mathbf{D}$ | Distance |
| $\mathbf{E}$ | $\mathrm{E}_{\mathrm{k}}$ East coordinate |
| $\mathbf{N}$ | $\mathrm{N}_{\mathrm{k}}$ North coordinate |
| $\mathbf{X}$ | $\mathrm{N}_{\mathrm{i}}$ North coordinate of Instrument Point |
| $\mathbf{Y}$ | $\mathrm{E}_{\mathrm{i}}$ East coordinate of Instrument Point |

## PROGRAM LENGTH AND CHECKSUM

## LN = 112; CK = A366

$\star$ Length \& Checksum: $\rightarrow 2 \rightarrow 2$ ENTER (Hold)

## PROGRAM NOTES

Flag 1 is used to test to see if new point is to be next Instrument Point.


The calculator must contain LBL Z which contains the Polar to Rectangular routines
XEQ Z002 on line $J 025$ is the Rectangular $\rightarrow$ Polar conversion

## USER INSTRUCTIONS <br> OFFSETS PROGRAM

1. To start program press $\mathrm{XEQ} \mathbf{O 0 0 1}$
2. Display B? Enter: $B_{1}$, the bearing of traverse line 1; then press R/S
1.0000
$\begin{array}{llll}\text { 3. } & \text { Display } & \text { B? } \\ 2.0000\end{array} \quad$ Enter: $\quad B_{2}$, the bearing of traverse line 2 ; then press $R / \mathbf{S}$
3. Display D? Enter: $\pm d_{1}$, the offset from traverse line 1 ; then press R/S
4. Display $\begin{array}{ll}\text { D? } \\ 22.0000\end{array} \quad$ Enter: $\pm d_{2}$, the offset from traverse line 2; then press R/S
5. Radiation Bearing (D.MMSS) $B_{3}$ and Distance $d_{3}$ displayed at successive R/S. GoTo step 2 .

Rule: Offset distances are $\left\{\begin{array}{c}\text { positive } \\ \text { negative }\end{array}\right\}$ if point is to the $\left\{\begin{array}{c}\text { right } \\ \text { left }\end{array}\right\}$ of the traverse line looking in the direction of the bearing.

Derivation of formula: Radiation from offsets

$$
\begin{aligned}
d_{1} \tan (90-\theta)=\frac{d_{1}}{\tan \theta} & \begin{aligned}
\tan \alpha & =\frac{d_{1}}{\frac{d_{1}}{\tan \theta}+\frac{d_{2}}{\sin \theta}} \\
& =\frac{d_{1} \sin \theta}{d_{1} \cos \theta+d_{2}} \\
& =\frac{\sin \theta}{\cos \theta+\frac{d_{2}}{d_{1}}}
\end{aligned}
\end{aligned}
$$

Conventions: $\quad \theta=B_{2}-B_{1}$

$$
\begin{aligned}
& d \text { is }\left\{\begin{array}{l}
+ \text { tve } \\
-t^{\text {tve }}
\end{array}\right\} \text { if point is }\left\{\begin{array}{c}
\text { right } \\
\text { left }
\end{array}\right\} \text { of line } \\
& B_{3}=B_{1}+\alpha
\end{aligned}
$$

Formula: $\quad \tan \alpha=\frac{\sin \theta}{\cos \theta-\frac{d_{2}}{d_{1}}}$

| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0001 | LBL 0 |  |  |  |  |
| 0002 | 1 |  |  |  |  |
| 0003 | STO B | 1 |  |  |  |
| 0004 | INPUT B | Enter Bearing of 1st line ( $\mathrm{B}_{1}$ ) |  |  |  |
| 0005 | HMS $\rightarrow$ | $\mathrm{B}_{1}$ |  |  |  |
| 0006 | STO A |  |  |  |  |
| 0007 | 2 |  |  |  |  |
| 0008 | STO B | 2 |  |  |  |
| 0009 | INPUT B | Enter Bearing of 2nd line ( $\mathrm{B}_{2}$ ) |  |  |  |
| 0010 | HMS $\rightarrow$ | $\mathrm{B}_{2}$ |  |  |  |
| 0011 | RCL A | $\mathrm{B}_{1}$ | $\mathrm{B}_{2}$ |  |  |
| 0012 | - | $\pm \theta=\mathrm{B}_{2}-\mathrm{B}_{1}$ |  |  |  |
| 0013 | 360 | 360 | $\pm \theta$ |  |  |
| 0014 | $x<>y$ | $\pm \theta$ | 360 |  |  |
| 0015 | $x<0$ ? |  |  |  |  |
| 0016 | + |  |  |  |  |
| 0017 | ST0 T | $\theta$ |  |  |  |
| 0018 | 11 |  |  |  |  |
| 0019 | STO D | 11 |  |  |  |
| 0020 | INPUT D | Enter Offset $\pm \mathrm{d}_{1}$ from 1st line |  |  |  |
| 0021 | STO C | $\pm \mathrm{d}_{1}$ |  |  |  |
| 0022 | 22 |  |  |  |  |
| 0023 | STO D | 22 |  |  |  |
| 0024 | INPUT D | Enter Offset $\pm \mathrm{d}_{2}$ from 2nd line |  |  |  |
| 0025 | RCL T | $\theta$ |  |  |  |
| 0026 | SIN | $\sin (\theta)$ | $\pm \mathrm{d}_{2}$ |  |  |
| 0027 | RCL T | $\theta$ | $\sin (\theta)$ | $\pm \mathrm{d}_{2}$ |  |
| 0028 | COS | $\cos (\theta)$ | $\sin (\theta)$ | $\pm \mathrm{d}_{2}$ |  |
| 0029 | RCL D | $\pm \mathrm{d}_{2}$ | $\cos (\theta)$ | $\sin (\theta)$ | $\pm \mathrm{d}_{2}$ |
| 0030 | RCL C | $\pm \mathrm{d}_{1}$ | $\pm \mathrm{d}_{2}$ | $\cos (\theta)$ | $\sin (\theta)$ |
| 0031 | $\div$ | $\pm \mathrm{d}_{2} / \mathrm{d}_{1}$ | $\cos (\theta)$ | $\sin (\theta)$ | $\sin (\theta)$ |
| 0032 | - | $\cos \theta-\left(\mathrm{d}_{2} / \mathrm{d}_{1}\right)$ | $\sin (\theta)$ | $\sin (\theta)$ | $\sin (\theta)$ |
| 0033 | $\div$ | $\tan (\alpha)$ |  |  |  |
| 0034 | ATAN | $\pm \alpha$ |  |  |  |
| 0035 | STO+A | $\mathrm{B}_{3}$ |  |  |  |
| 0036 | SIN | sin( $\alpha$ ) |  |  |  |
| 0037 | STO -C |  |  |  |  |
| 0038 | RCL C | $\pm \mathrm{d}_{3}$ |  |  |  |
| 0039 | 0 | 0 | $\pm \mathrm{d}_{3}$ |  |  |
| 0040 | $x>y$ ? |  |  |  |  |
| 0041 | 180 |  |  |  |  |
| 0042 | STO+A |  |  |  |  |


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0043 | RCL A | $\mathrm{B}_{3}$ |  |  |  |
| 0044 | 360 | 360 | $\mathrm{B}_{3}$ |  |  |
| 0045 | $x<y$ ? |  |  |  |  |
| 0046 | STO-A |  |  |  |  |
| 0047 | RCL A | $\mathrm{B}_{3}$ |  |  |  |
| 0048 | $\rightarrow$ HMS |  |  |  |  |
| 0049 | STO B |  |  |  |  |
| 0050 | VIEW B | Radia | Bearing |  |  |
| 0051 | RCL C | $\pm \mathrm{d}_{3}$ |  |  |  |
| 0052 | ABS |  |  |  |  |
| 0053 | STO D |  |  |  |  |
| 0054 | VIEW D | Radiation Distance $\mathrm{d}_{3}$ |  |  |  |
| 0055 | GTO 0002 |  |  |  |  |

## STORAGE REGISTERS

| $\mathbf{A}$ | $\mathrm{B}_{1} ; \mathrm{B}_{3}$ |
| :--- | :--- |
| $\mathbf{B}$ | $\mathrm{~B}_{2}(\mathrm{D}$. MMSS $) ; \mathrm{B}_{3}(\mathrm{D}$. MMSS $)$ |
| $\mathbf{C}$ | $\pm \mathrm{d}_{1} ; \pm \mathrm{d}_{3}$ |
| $\mathbf{D}$ | $\pm \mathrm{d}_{2} ; \mathrm{d}_{3}$ |
| $\mathbf{T}$ | $\theta$ |
|  |  |

## PROGRAM LENGTH AND CHECKSUM

## $L N=181 ; C K=A 802$



## USER INSTRUCTIONS <br> RESECTION PROGRAM <br> (Auxiliary angles method)

1. To start program press XEQ S001
2. Display E? Enter: East coordinate of Point $1\left(P_{1}\right)$; then press R/S
1.0000
3. Display N ? Enter: North coordinate of $\mathrm{P}_{1}$; then press R/S
4. Display E? Enter: East coordinate of $\mathrm{P}_{2}$; then press R/S
5. 2.0000

Enter: North coordinate of $P_{2}$; then press $R / S$
2.0000
6. Display E? Enter: East coordinate of $\mathrm{P}_{3}$; then press R/S 3.0000
7. Display N ? 3.0000
8. Display A? 0.0000
$\begin{array}{ll}\text { 9. } & \text { Display } \\ & B ? \\ & 0.0000\end{array}$
Enter: North coordinate of $P_{3}$; then press $R / S$
Enter: Angle $\alpha$ (D.MMSS) at Resection Point P; then press R/S
10. East and North coordinate of Resection Point displayed at successive R/S. GoTo step 2.

Notes: (1) Coordinates of points $\mathrm{P}_{1}, \mathrm{P}_{2}, \mathrm{P}_{3}$ must be entered left to right (clockwise direction) as seen from the Resection Point $P$.
(2) Observed angles $\alpha$ and $\beta$ are angles $\mathrm{P}_{1}-\mathrm{P}-\mathrm{P}_{2}$ and $\mathrm{P}_{2}-\mathrm{P}-\mathrm{P}_{3}$ respectively.

## EXAMPLE



## RESECTION - AUXILIARY ANGLES



CASE 1

CASE 2


GIVEN: $\quad 1\left(\mathrm{E}_{1}, \mathrm{~N}_{1}\right), 2\left(\mathrm{E}_{2}, \mathrm{~N}_{2}\right), 3\left(\mathrm{E}_{3}, \mathrm{~N}_{3}\right)$
OBSERVED: $\quad \alpha, \beta$
COMPUTE: $\quad \boldsymbol{P}\left(\mathrm{E}_{\mathrm{P}}, \mathrm{N}_{\mathrm{P}}\right)$

1. Compute bearings and distances of lines $2-1$ and $2-3$
2. Calculate angle $\gamma$ as the difference between bearings $B_{21}$ and $B_{23}$. [ $B_{\mathrm{KJ}}$ means the bearing from K to J ]
3. 

$$
\begin{equation*}
\varphi+\psi=360^{\circ}-(\alpha+\beta+\gamma)=\theta \tag{1}
\end{equation*}
$$

4. From sine rule:

$$
\begin{array}{lll}
\frac{d_{2 p}}{\sin \varphi}=\frac{d_{21}}{\sin \alpha} & \text { or } & d_{2 P}=\frac{d_{21} \sin \varphi}{\sin \alpha} \\
\frac{d_{2 p}}{\sin \psi}=\frac{d_{23}}{\sin \beta} & \text { or } & d_{2 P}=\frac{d_{23} \sin \psi}{\sin \beta} \tag{3}
\end{array}
$$

Equating (2) and (3) gives

$$
\begin{equation*}
\frac{\sin \varphi}{\sin \psi}=\frac{d_{23} \sin \alpha}{d_{21} \sin \beta}=a \tag{4}
\end{equation*}
$$

5. From (4) $\sin \varphi=a \sin \psi$, but from (1) $\psi=\theta-\varphi$; hence $\sin \varphi=a \sin (\theta-\varphi)=a(\sin \theta \cos \varphi-\cos \theta \sin \varphi)$.

Dividing both sides by $\cos \varphi$ and re-arranging gives $\tan \varphi(1+a \cos \theta)=a \sin \theta$ and

$$
\begin{equation*}
\tan \varphi=\frac{a \sin \theta}{1+a \cos \theta} \tag{5}
\end{equation*}
$$

6. After computing $\theta$ [using (1)], $a$ [using (4)] and $\varphi$ [using (5)] then $\psi$ can be calculated using (1).
7. The bearing $B_{1 \mathrm{P}}$ (bearing of the line $1-P$ ) is given by

$$
\begin{equation*}
B_{1 P}=B_{12}+\varphi \tag{6}
\end{equation*}
$$

The distance $d 1 \mathrm{P}$ (distance of line $1-P$ ) is obtained using the sine rule in triangle $12 P$ and

$$
\begin{equation*}
d_{1 P}=\frac{d_{12} \sin (\alpha+\varphi)}{\sin \alpha} \tag{7}
\end{equation*}
$$

8. $E_{P}$ and $N_{P}$ obtained from $E_{1}$ and $N_{1}$ and the bearing $B_{1 P}$ and distance $d_{1 P}$ of the line $1-P$.

## EXAMPLE



| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S001 | LBL S |  |  |  |  |
| S002 | CLVARS |  | START RESECTION PROGRAM |  |  |
| S003 | 1 |  |  |  |  |
| S004 | XEQ S093 | Enter coordinates of Point 1 |  |  |  |
| S005 | RCL E | $\mathrm{E}_{1}$ |  |  |  |
| S006 | STO R | $\mathrm{E}_{1}$ |  |  |  |
| S007 | RCL N | $\mathrm{N}_{1}$ |  |  |  |
| S008 | STO U | $\mathrm{N}_{1}$ | $\mathrm{E}_{1}$ |  |  |
| S009 | 2 |  |  |  |  |
| S010 | XEQ S093 | Enter coordinates of Point 2 |  |  |  |
| S011 | RCL E | $\mathrm{E}_{2}$ |  |  |  |
| S012 | STO S | $\mathrm{E}_{2}$ |  |  |  |
| S013 | RCL N | $\mathrm{N}_{2}$ |  |  |  |
| S014 | ST0 V | $\mathrm{N}_{2}$ | $\mathrm{E}_{2}$ |  |  |
| S015 | 3 |  |  |  |  |
| S016 | XEQ S093 | Enter coordinates of Point 3 |  |  |  |
| S017 | INPUT A | Enter angle a (D.MMSS) |  |  |  |
| S018 | HMS $\rightarrow$ | $\alpha$ |  |  |  |
| S019 | STO A |  |  |  |  |
| S020 | INPUT B | Enter angle $\beta$ (D.MMSS) |  |  |  |
| S021 | HMS $\rightarrow$ |  |  |  |  |
| S022 | STO B | $\beta$ |  |  |  |
| S023 | RCL S | $\mathrm{E}_{2}$ |  |  |  |
| S024 | RCL E | $\mathrm{E}_{3}$ | $\mathrm{E}_{2}$ |  |  |
| S025 | - | $\Delta \mathrm{E}_{32}=\mathrm{E}_{2}-\mathrm{E}_{3}$ |  |  |  |
| S026 | RCL V | $\mathrm{N}_{2}$ | $\Delta \mathrm{E}_{32}$ |  |  |
| S027 | RCL N | $\mathrm{N}_{3}$ | $\mathrm{N}_{2}$ | $\Delta \mathrm{E}_{32}$ |  |
| S028 | - | $\Delta N_{32}=N_{2}-N_{3}$ | $\Delta \mathrm{E}_{32}$ |  |  |
| S029 | XEQ Z002 | $\mathrm{d}_{32}=\mathrm{d}_{23}$ | $\mathrm{B}_{32}$ |  |  |
| S030 | RCL A | $\alpha$ | $\mathrm{d}_{23}$ | $\mathrm{B}_{32}$ |  |
| S031 | SIN | $\sin (\alpha)$ | $\mathrm{d}_{23}$ | $\mathrm{B}_{32}$ |  |
| S032 | $\times$ | $\mathrm{d}_{23} \sin (\alpha)$ | $\mathrm{B}_{32}$ |  |  |
| S033 | ST0 C |  |  |  |  |
| S034 | $x<>y$ | $\mathrm{B}_{32}$ | $\mathrm{d}_{23} \sin (\alpha)$ |  |  |
| S035 | 180 | 180 | $\mathrm{B}_{32}$ | $\mathrm{d}_{23} \sin (\alpha)$ |  |
| S036 | + | $\mathrm{B}_{23}$ | $\mathrm{d}_{23} \sin (\alpha)$ |  |  |
| S037 | RCL S | $\mathrm{E}_{2}$ | $\mathrm{B}_{32}$ | $\mathrm{d}_{23} \sin (\alpha)$ |  |
| S038 | RCL R | $\mathrm{E}_{1}$ | $\mathrm{E}_{2}$ | $\mathrm{B}_{32}$ | $\mathrm{d}_{23} \sin (\alpha)$ |
| S039 | - | $\Delta \mathrm{E}_{12}=\mathrm{E}_{2}-\mathrm{E}_{1}$ | $\mathrm{B}_{32}$ | $\mathrm{d}_{23} \sin (\alpha)$ | $\mathrm{d}_{23} \sin (\alpha)$ |
| S040 | RCL V | $\mathrm{N}_{2}$ |  |  |  |
| S041 | RCL U | $\mathrm{N}_{1}$ | $\mathrm{N}_{2}$ | $\Delta \mathrm{E}_{12}$ | $\mathrm{B}_{32}$ |
| S042 | - | $\Delta N_{12}=N_{2}-\mathrm{N}_{1}$ | $\Delta \mathrm{E}_{12}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S043 | XEQ Z002 | $\mathrm{d}_{12}$ | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S044 | STO D |  |  |  |  |
| S045 | RCL B | $\beta$ | $\mathrm{d}_{12}$ | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ |
| S046 | SIN | sin ( $\beta$ ) | $\mathrm{d}_{12}$ | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ |
| S047 | $\times$ | $\mathrm{d}_{12} \sin (\beta)$ | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S048 | STO -C |  |  |  |  |
| S049 | R $\downarrow$ | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ | $\mathrm{d}_{12} \sin (\beta)$ |
| S050 | STO F |  |  |  |  |
| S051 | 180 | 180 | $\mathrm{B}_{12}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S052 | + | $\mathrm{B}_{21}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S053 | $x<>y$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{21}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S054 | - | $\pm \gamma$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S055 | 360 |  |  |  |  |
| S056 | $x<>y$ | $\pm \gamma$ | 360 | $\mathrm{B}_{32}$ | $\mathrm{B}_{32}$ |
| S057 | $x<0$ ? |  |  |  |  |
| S058 | + | $\gamma$ |  |  |  |
| S059 | RCL A | $\alpha$ | $\gamma$ |  |  |
| S060 | RCL B | $\beta$ | $\alpha$ | $\gamma$ |  |
| S061 | + |  |  |  |  |
| S062 | + | $\alpha+\beta+\gamma$ |  |  |  |
| S063 | 360 |  |  |  |  |
| S064 | $x<>y$ |  |  |  |  |
| S065 | - | $\theta=360-(\alpha+\beta$ | + $\gamma$ ) |  |  |
| S066 | RCL C | a | $\theta$ |  |  |
| S067 | XEQ Z015 | $a \times \cos (\theta)$ | $\mathrm{a} \times \sin (\theta)$ |  |  |
| S068 | 1 |  |  |  |  |
| S069 | + | $1+\operatorname{acos}(\theta)$ | asin( $\theta$ ) |  |  |
| S070 | XEQ Z002 |  | $\varphi$ |  |  |
| S071 | $x<>y$ | $\varphi$ |  |  |  |
| S072 | STO+F | $\varphi$ |  |  |  |
| S073 | RCL A | $\alpha$ | $\varphi$ |  |  |
| S074 | + | $\alpha+\varphi$ |  |  |  |
| S075 | SIN | $\sin (\alpha+\varphi)$ |  |  |  |
| S076 | STO $\times$ D |  |  |  |  |
| S077 | RCL A | $\alpha$ |  |  |  |
| S078 | SIN | sin( $\alpha$ ) |  |  |  |
| S079 | STO -D |  |  |  |  |
| S080 | RCL F | $\mathrm{B}_{1} \mathrm{P}$ |  |  |  |
| S081 | RCL D | $\mathrm{d}_{1 \mathrm{p}}$ | $\mathrm{B}_{1 \mathrm{p}}$ |  |  |
| S082 | XEQ Z015 | $\Delta \mathrm{N}_{1} \mathrm{P}$ | $\Delta \mathrm{E}_{1 \mathrm{P}}$ |  |  |



## STORAGE REGISTERS

| $\mathbf{A}$ | $\alpha$ |
| :--- | :--- |
| $\mathbf{B}$ | $\beta$ |
| $\mathbf{C}$ | $\mathrm{d}_{23} \sin (\alpha) ; \mathrm{a}=\mathrm{d}_{23} \sin (\alpha) / \mathrm{d}_{12} \sin (\beta)$ |
| $\mathbf{D}$ | $\mathrm{d}_{12} ; \mathrm{d}_{12} \sin (\alpha+\varphi) ; \mathrm{d}_{1 \mathrm{P}}=\left[\mathrm{d}_{12} \sin (\alpha+\varphi)\right] / \sin (\alpha)$ |
| $\mathbf{F}$ | $\mathrm{B}_{12} ; \mathrm{B}_{1 \mathrm{P}}=\left(\mathrm{B}_{12}+\varphi\right)$ |
| $\mathbf{E}$ | $\mathrm{E}_{\mathrm{k}} ; \mathrm{E}_{3} ; \mathrm{E}_{\mathrm{P}}$ |
| $\mathbf{N}$ | $\mathrm{N}_{\mathrm{k}} ; \mathrm{N}_{3} ; \mathrm{N}_{\mathrm{P}}$ |
| $\mathbf{R}$ | $\mathrm{E}_{1}$ |
| $\mathbf{U}$ | $\mathrm{~N}_{1}$ |
| $\mathbf{S}$ | $\mathrm{E}_{2}$ |
| $\mathbf{V}$ | $\mathrm{~N}_{2}$ |

## PROGRAM LENGTH AND CHECKSUM

$L N=307 ; C K=A E 78$


## PROGRAM NOTES

$P_{1}, P_{2}, P_{3}$ means Points 1, 2 and 3.
$E_{1}, E_{2}, ~ e t c$. and $N_{1}, N_{2}$, etc. mean east and north coordinates of $P_{1}, P_{2}$, etc.
$\Delta E_{12}=E_{2}-E_{1}, \quad \Delta N_{12}=N_{2}-N_{1}, \quad$ etc.
$B_{12}$ means bearing of the line from $P_{1}$ to $P_{2}$
$d_{12}$ means distance from $P_{1}$ to $P_{2}$

|  | S001 to S016 | Initialisation; storing coordinates $P_{1}, P_{2}, P_{3}$. |
| :---: | :---: | :---: |
| Lines | S017 to S022 | Entering and storing observed angles $\alpha$ and $\beta$ at the Resection Point $P$. |
| Lines | S023 to S029 | Bearing and distance $\mathrm{P}_{3}$ to $\mathrm{P}_{2}$. |
| Lines | S034 to S036 | Note here that $\mathrm{B}_{23}=\mathrm{B}_{32}+180^{\circ}$ |
| Lines | S037 to S043 | Bearing and distance $\mathrm{P}_{1}$ to $\mathrm{P}_{2}$. |
| Lines | S044 to S065 | Calculation of angles $\gamma$ at $P_{2}$ and $\theta=360^{\circ}-(\alpha+\beta+\gamma)$; and the ratio $a=d_{23} \sin (\alpha) / d_{12} \sin (\beta)$ |
| Lines | S066 to S079 | Calculation of auxiliary angle $\varphi$ and the bearing and distance $P_{1}$ to the resection Point $P$ : $B_{1 p}=\left(B_{12}+\varphi\right)$ and $d_{1 p}=\left[d_{12} \sin (\alpha+\varphi)\right] / \sin (\alpha)$. |
| Lines | S080 to S091 | Calculation and display of coordinates of Resected Point P. |
| Lines | S093 to S097 | Subroutine for entering coordinates of $P_{1}, P_{2}, P_{3}$. |

The calculator must contain LBL Z which contains the Polar to Rectangular routines

XEQ Z002 on lines S067, S082 is the Rectangular $\rightarrow$ Polar conversion
XEQ Z015 on lines S029, S043, S070 is the Polar $\rightarrow$ Rectangular conversion

## USER INSTRUCTIONS TRAVERSE ADJUSTMENT PROGRAM

This program can perform either a BOWDITCH ${ }^{1}$ or a CRANDALL ${ }^{2}$ adjustment on a closed traverse (or figure). The bearings and distances of each line of the closed traverse must be entered before selecting the method of adjustment ( $1=$ Bowditch; $2=$ Crandall $)$.
After all lines have been entered and adjustment type selected the program will display the adjusted bearings and distances and then the area of the adjusted figure.
A closed traverse must start and end at known points (east and north coordinates known); but in the case of a loop traverse the start and end points will be the same. The program requires that $\mathrm{D}_{\mathrm{E}}=\mathrm{E}_{\mathrm{END}}-\mathrm{E}_{\mathrm{START}}$ and $\mathrm{D}_{\mathrm{N}}=\mathrm{N}_{\mathrm{END}}-\mathrm{N}_{\mathrm{START}}$ are known. If the traverse is a loop traverse $\mathrm{D}_{\mathrm{E}}=\mathrm{D}_{\mathrm{N}}=0$

1. To start program press XEQ A001
2. Display B? Enter: Bearing (D.MMSS); then press R/S
$\begin{array}{lll} & 0.0000 & \text { [Bearing of lines that are } 0^{\circ} 00^{\prime} 00^{\prime \prime} \text { must be entered as } 360^{\circ} 00^{\prime} 00^{\prime \prime} \text { ] } \\ \text { 3. Display } & \text { D? } & \text { Enter: Distance; then press R/S } \\ 0.0000 & & \end{array}$
3. Repeat steps 2 and 3 until all known information is entered; then enter 0 at the Bearing prompt and 0 at the Distance prompt (just press $\mathrm{R} / \mathrm{S}$ at the prompts)
$\begin{array}{lll}\text { 5. Display } & X \text { ? } & \text { Enter: } D_{E} \text {; then press } R / S \\ & 0.0000 & \text { [If loop traverse } D_{E}=0 \text {, just press R/S] }\end{array}$
$\begin{array}{lll}\text { 6. Display } & \text { Y? } & \text { Enter: } D_{N} \text {; then press R/S } \\ & 0.0000 & \text { [If loop traverse } D_{N}=0 \text {, just press R/S] }\end{array}$
4. Display F? Enter: $1=$ BOWDITCH or $2=$ CRANDALL; then press R/S
0.0000
5. Adjusted Bearings (D.MMSS) and adjusted Distances displayed at successive R/S.
[Note that Crandall's adjustment only adjusts distances]
6. Adjusted Area displayed at last prompt. Press R/S and go to step 2 for new adjustment.
[^0]
## THEORY AND FORMULA

Theory, formula and examples of Bowditch's and Crandall's adjustments can be found in Notes on Least Squares, Geospatial Science, RMIT University, Chapter 6, pp.6-15-6-26. A summary of the formula and the sequence of computation is presented below.

## BOWDITCH

A closed traverse of $k=1,2,3 \ldots, n$ lines, sides or legs having bearings $\phi_{k}$ and distances $d_{k}$ (or a figure of $n$ sides) that has a misclosure may be adjusted in the following manner.

1. Each traverse line (having bearing and distance) has east and north components $\Delta E_{k}=d_{k} \sin \phi_{k}, \Delta N_{k}=d_{k} \cos \phi_{k}$, and the sums of these components for the traverse are $S_{E}=\sum_{k=1}^{n} \Delta E_{k}$ and $S_{N}=\sum_{k=1}^{n} \Delta N_{k}$
2. A traverse has a total length $L=\sum_{k=1}^{n} d_{k}$
3. A closed traverse has a start point and an end point assumed to have known east and north coordinates; $E_{\text {START }}, N_{\text {START }}, E_{\text {END }}, N_{\text {END }}$ and differences; $D_{E}=E_{\text {END }}-E_{\text {START }}$ and $D_{N}=N_{\text {END }}-N_{\text {START }}$. If the traverse is a loop traverse (starting and ending at the same point), then $D_{E}=D_{N}=0$.
4. The east and north components of each traverse leg may be adjusted by adding corrections $d E_{k}=d_{k}\left(\frac{D_{E}-S_{E}}{L}\right)$ and $d N_{k}=d_{k}\left(\frac{D_{N}-S_{N}}{L}\right)$ so that $\left\{\begin{array}{l}\Delta E_{k} \\ \Delta N_{k}\end{array}\right\}_{\text {ADJUST }}=\left\{\begin{array}{l}\Delta E_{k} \\ \Delta N_{k}\end{array}\right\}_{O B S}+\left\{\begin{array}{l}d E_{k} \\ d N_{k}\end{array}\right\}$
5. Adjusted bearings and distances and area are then computed from the adjusted east and north components.

## CRANDALL

A closed traverse of $k=1,2,3 \ldots, n$ lines, sides or legs having bearings $\phi_{k}$ and distances $d_{k}$ (or a figure of $n$ sides) that has a misclosure may be adjusted in the following manner.

1. First adjust the bearings of the traverse so that they close perfectly. This may be an arbitrary adjustment.
2. Each traverse line (having bearing and distance) has east and north components $\Delta E_{k}=d_{k} \sin \phi_{k}, \Delta N_{k}=d_{k} \cos \phi_{k}$, and the sums of these components for the traverse are $S_{E}=\sum_{k=1}^{n} \Delta E_{k}$ and $S_{N}=\sum_{k=1}^{n} \Delta N_{k}$

## THEORY AND FORMULA continued

3. In addition, the traverse has the following summations: $a=\sum_{k=1}^{n} \frac{\left(\Delta E_{k}\right)^{2}}{d_{k}}, b=\sum_{k=1}^{n} \frac{\left(\Delta N_{k}\right)^{2}}{d_{k}}$, and $c=\sum_{k=1}^{n} \frac{\Delta E_{k} \Delta N_{k}}{d_{k}}$
4. A closed traverse has a start point and an end point assumed to have known east and north coordinates; $E_{\text {START }}, N_{\text {START }}, E_{\text {END }}, N_{\text {END }}$ and differences; $D_{E}=E_{\text {END }}-E_{\text {START }}$ and $D_{N}=N_{\text {END }}-N_{\text {START }}$. If the traverse is a loop traverse (starting and ending at the same point), then $D_{E}=D_{N}=0$.
5. Two 'multipliers' are computed: $\left\{\begin{array}{l}k_{1} \\ k_{2}\end{array}\right\}=\left\{\begin{array}{l}\frac{b\left(D_{E}-S_{E}\right)-c\left(D_{N}-S_{N}\right)}{a b-c^{2}} \\ \frac{a\left(D_{N}-S_{N}\right)-c\left(D_{E}-S_{E}\right)}{a b-c^{2}}\end{array}\right\}$
6. A residual $v_{k}$ for each traverse line is computed from $v_{k}=k_{1} \Delta E_{k}+k_{2} \Delta N_{k}$ and added to the observed traverse distance to obtain the adjusted traverse distance: $d_{\text {ADJUST }}=d_{\text {OBS }}+v$

## EXAMPLE 1



Figure 1. Fieldnotes of traverse
Figure 1 shows a traverse between points $A, B, C$ and $D$. The bearing datum of the survey is the line $A B 285^{\circ} 00^{\prime} 00^{\prime \prime}$. The distances are horizontal distances. Observed face-left (FL) bearings are shown along the traverse line and the seconds part of the face-right (FR) bearing is shown above. The mean of the FL/FR seconds is shown to the right of the brace \}. The angular misclose in the traverse is 20 ", which is revealed in the forward and reverse bearings on the line $C D$.

## 1. BOWDITCH ADJUSTMENT OF EXAMPLE 1

For the purpose of the exercise we assume that the angular misclose of 20 " is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

| Line | Bearing | Distance | components |  | corrections |  | adjusted components |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ | $d E_{k}$ | $d N_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ |
| $1: A B$ | $285^{\circ} 00^{\prime} 00^{\prime \prime}$ | 268.786 | -259.6273 | 69.5669 | 0.0068 | -0.0139 | -259.6205 | 69.5530 |
| $2: B C$ | $346^{\circ} 37^{\prime} 29^{\prime \prime}$ | 156.627 | -36.2322 | 152.3786 | 0.0040 | -0.0081 | -36.2282 | 152.3705 |
| $3: C D$ | $93^{\circ} 42^{\prime} 25^{\prime \prime}$ | 148.650 | 148.3390 | -9.6107 | 0.0038 | -0.0077 | 148.3428 | -9.6184 |
| $4: D A$ | $145^{\circ} 12^{\prime} 31^{\prime \prime}$ | 258.503 | 147.4993 | -212.2917 | 0.0066 | -0.0134 | 147.5059 | -212.3051 |
|  | sums | 832.5660 | -0.0212 | 0.0431 | 0.0212 | -0.0431 | 0.0000 | 0.0000 |

$$
L=\sum_{k=1}^{n} d_{k}=832.5660, S_{E}=\sum_{k=1}^{n} \Delta E_{k}=-0.0212 \text { and } S_{N}=\sum_{k=1}^{n} \Delta N_{k}=0.0431
$$

Since this is a loop traverse $D_{E}=D_{N}=0$ and $D_{E}-S_{E}=0.0212, D_{N}-S_{N}=-0.0431$
The corrections to the traverse components are: $\quad d E_{k}=d_{k}\left(\frac{D_{E}-S_{E}}{L}\right)=d_{k}\left(\frac{0.0212}{832.5660}\right)$

$$
d E_{k}=d_{k}\left(\frac{D_{N}-S_{N}}{L}\right)=d_{k}\left(\frac{-0.0431}{832.5660}\right)
$$

The adjusted traverse is

| Line | Bearing | Distance |
| :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ |
| $1: A B$ | $284^{\circ} 59^{\prime} 51^{\prime \prime}$ | 268.7758 |
| 2: $B C$ | $34^{\circ} 37^{\prime} 32^{\prime \prime}$ | 156.6182 |
| $3: C D$ | $93^{\circ} 42^{\prime} 35^{\prime \prime}$ | 148.6543 |
| $4: D A$ | $145^{\circ} 12^{\prime} 33^{\prime \prime}$ | 258.5178 |

Using the program: press XEQ A001 (or XEQ A ENTER )
Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry
When all sides have been keyed in, enter 0 at the prompt B ? and press $\mathrm{R} / \mathrm{S}$; and 0 at the prompt D ? and press R/S (or simply press R/S at both prompts).

At the prompt X ? enter 0 and press $\mathrm{R} / \mathrm{S}\left(\mathrm{D}_{E}=0\right)$ and at the prompt Y ? enter 0 and press $\mathrm{R} / \mathrm{S}$ ( $D_{N}=0$ )

At the prompt F? enter 1 and press R/S.
The calculator will then display the adjusted bearing at $\mathrm{B}=$. Press $\mathrm{R} / \mathrm{S}$ and the adjusted distance will be displayed at $\mathrm{D}=$. Repeat pressing of $\mathrm{R} / \mathrm{S}$ will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $\mathrm{A}=$ (The area $=-33,556.9387 \mathrm{~m}^{2}$ )

## 2. CRANDALL ADJUSTMENT OF EXAMPLE 1

For the purpose of the exercise we assume that the angular misclose of 20 " is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

| Line | Bearing | Distance | components |  |  | $\left(\Delta E_{k}\right)^{2}$ |  | $\left(\Delta N_{k}\right)^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d_{k}$ |  | $\Delta E_{k} \Delta N_{k}$ | residual |  |  |  |  |  |
| $k$ | $\phi_{k}$ | $d_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ | $\frac{d_{k}}{}$ | $\frac{d_{k}}{}$ | $v_{k}$ |  |
| $1: A B$ | $285^{\circ} 00^{\prime} 00^{\prime \prime}$ | 268.786 | -259.6273 | 69.5669 | 250.7808 | 18.0052 | -67.1965 | -0.004 |
| $2: B C$ | $346^{\circ} 37^{\prime} 29^{\prime \prime}$ | 156.627 | -36.2322 | 152.3786 | 8.3815 | 148.2455 | -35.2495 | -0.021 |
| $3: C D$ | $93^{\circ} 42^{\prime} 25^{\prime \prime}$ | 148.650 | 148.3390 | -9.6107 | 148.0286 | 0.6214 | -9.5906 | -0.002 |
| $4: D A$ | $145^{\circ} 12^{\prime} 31^{\prime \prime}$ | 258.503 | 147.4993 | -212.2917 | 84.1616 | 174.3414 | -121.1316 | 0.027 |

$S_{E}=\sum_{k=1}^{n} \Delta E_{k}=-0.0212, S_{N}=\sum_{k=1}^{n} \Delta N_{k}=0.0431$
$a=\sum_{k=1}^{n} \frac{\left(\Delta E_{k}\right)^{2}}{d_{k}}=491.3526, b=\sum_{k=1}^{n} \frac{\left(\Delta N_{k}\right)^{2}}{d_{k}}=341.2134$ and $c=\sum_{k=1}^{n} \frac{\Delta E_{k} \Delta N_{k}}{d_{k}}=-233.1681$

Since this is a loop traverse $D_{E}=D_{N}=0$ and $D_{E}-S_{E}=0.0212, D_{N}-S_{N}=-0.0431$
The multipliers are: $\quad k_{1}=\frac{b\left(D_{E}-S_{E}\right)-c\left(D_{N}-S_{N}\right)}{a b-c^{2}}=-2.4593 e-05$

$$
k_{2}=\frac{a\left(D_{N}-S_{N}\right)-c\left(D_{E}-S_{E}\right)}{a b-c^{2}}=-1.4324 e-04
$$

The residuals are:

$$
v_{k}=k_{1} \Delta E_{k}+k_{2} \Delta N_{k}
$$

The adjusted traverse (nearest mm) is

| Line | Bearing | Distance |
| :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ |
| $1: A B$ | $285^{\circ} 00^{\prime} 00^{\prime \prime}$ | 268.782 |
| $2: B C$ | $346^{\circ} 37^{\prime} 29^{\prime \prime}$ | 156.606 |
| $3: C D$ | $93^{\circ} 42^{\prime} 25^{\prime \prime}$ | 148.648 |
| $4: D A$ | $145^{\circ} 12^{\prime} 31^{\prime \prime}$ | 258.530 |

Using the program: press XEQ A001 (or XEQ A ENTER )
Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry
When all sides have been keyed in, enter 0 at the prompt B ? and press $\mathrm{R} / \mathrm{S}$; and 0 at the prompt D ? and press R/S (or simply press R/S at both prompts).

At the prompt X ? enter 0 and press $\mathrm{R} / \mathrm{S}\left(\mathrm{D}_{E}=0\right)$ and at the prompt Y ? enter 0 and press $\mathrm{R} / \mathrm{S}$ ( $D_{N}=0$ )

At the prompt F? enter 2 and press R/S.
The calculator will then display the adjusted bearing at $\mathrm{B}=$. Press $\mathrm{R} / \mathrm{S}$ and the adjusted distance will be displayed at $\mathrm{D}=$. Repeat pressing of $\mathrm{R} / \mathrm{S}$ will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $\mathrm{A}=$ $\left(\right.$ The area $\left.=-33,555.9331 \mathrm{~m}^{2}\right)$

## EXAMPLE 2



Figure 2 Traverse diagram showing field measurements, derived values and fixed values.
Figure 2 is a schematic diagram of a traverse run between two fixed stations $A$ and $B$ and oriented at both ends by angular observations to a third fixed station $C$.

The bearings of traverse lines shown on the diagram, unless otherwise indicated, are called "observed" bearings and have been derived from the measured angles (which have been derived from observed theodolite directions) and the fixed bearing $A C$. The difference between the observed and fixed bearings of the line $B C$ represents the angular misclose of $15^{\prime \prime}$. The coordinates of the traverse points $D, E$ and $F$ have been calculated using the observed bearings and distances and the fixed coordinates of $A$. The difference between the observed and fixed coordinates at $B$ represents a traverse misclosure.

## 3. BOWDITCH ADJUSTMENT OF EXAMPLE 2

For the purpose of the exercise we assume that the angular misclose of $15^{\prime \prime}$ is acceptable and that this error is apportioned equally at the five traverse points giving the observed traverse to be adjusted as shown in the left-columns of the table below

| Line | Bearing | Distance | components |  | corrections |  | adjusted components |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ | $d E_{k}$ | $d N_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ |
| $1: A D$ | $110^{\circ} 15^{\prime} 17^{\prime \prime}$ | 2401.609 | 2253.1002 | -831.4235 | -0.0029 | -0.0994 | 2253.0973 | -831.5229 |
| $2: D E$ | $68^{\circ} 34^{\prime} 12^{\prime \prime}$ | 1032.340 | 960.9688 | 377.1801 | -0.0012 | -0.0427 | 960.9676 | 377.1374 |
| $3: E F$ | $163^{\circ} 03^{\prime} 23^{\prime \prime}$ | 559.022 | 162.9160 | -534.7560 | -0.0007 | -0.0231 | 162.9153 | -534.7791 |
| $4: F B$ | $113^{\circ} 49^{\prime} 38^{\prime \prime}$ | 1564.683 | 1431.3217 | -632.1006 | -0.0019 | -0.0648 | 1431.3198 | -632.1654 |
|  | sums | 5557.6540 | 4808.3067 | -1621.1000 | -0.0067 | -0.2300 | 0.0000 | 0.0000 |

$L=\sum_{k=1}^{n} d_{k}=5557.65400, S_{E}=\sum_{k=1}^{n} \Delta E_{k}=4808.3067$ and $S_{N}=\sum_{k=1}^{n} \Delta N_{k}=-1621.1000$
$D_{E}=E_{\text {END }}-E_{\text {START }}=6843.085-2034.785=4808.300$
$D_{N}=N_{\text {END }}-N_{\text {START }}=7154.700-8776.030=-1621.330$
$D_{E}-S_{E}=-0.0067, D_{N}-S_{N}=-0.2300$
The corrections to the traverse components are: $\quad d E_{k}=d_{k}\left(\frac{D_{E}-S_{E}}{L}\right)=d_{k}\left(\frac{-0.0067}{5557.6540}\right)$

$$
d E_{k}=d_{k}\left(\frac{D_{N}-S_{N}}{L}\right)=d_{k}\left(\frac{-0.2300}{5557.6540}\right)
$$

The adjusted traverse is

| Line | Bearing | Distance |
| :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ |
| $1: A D$ | $110^{\circ} 15^{\prime} 25^{\prime \prime}$ | 2401.6407 |
| 2: $D E$ | $68^{\circ} 34^{\prime} 20^{\prime \prime}$ | 1032.3232 |
| 3: $E F$ | $163^{\circ} 03^{\prime} 26^{\prime \prime}$ | 559.0439 |
| $4: F B$ | $113^{\circ} 49^{\prime} 46^{\prime \prime}$ | 1564.7074 |

Using the program: press XEQ A001 (or XEQ $\mathbf{A}$ ENTER )
Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry
When all sides have been keyed in, enter 0 at the prompt B ? and press R/S; and 0 at the prompt D ? and press R/S (or simply press R/S at both prompts).
At the prompt $X$ ? enter 4808.300 and press $\mathrm{R} / \mathrm{S}\left(D_{E}=4808.300\right)$
At the prompt Y? enter -1621.330 and press R/S ( $D_{N}=-1621.330$ )
At the prompt F? enter 1 and press R/S.
The calculator will then display the adjusted bearing at $\mathrm{B}=$. Press R/S and the adjusted distance will be displayed at $\mathrm{D}=$. Repeat pressing of $\mathrm{R} / \mathrm{S}$ will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $\mathrm{A}=$ (The area $=-357,496.7606 \mathrm{~m}^{2}$ but is meaningless since this is not a closed polygon)

## 4. CRANDALL ADJUSTMENT OF EXAMPLE 2

For the purpose of the exercise we assume that the angular misclose of $20^{\prime \prime}$ is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

| Line | Bearing | Distance | components |  | $\frac{\left(\Delta E_{k}\right)^{2}}{d_{k}}$ | $\frac{\left(\Delta N_{k}\right)^{2}}{d_{k}}$ | $\frac{\Delta E_{k} \Delta N_{k}}{d_{k}}$ | resid. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ | $\Delta E_{k}$ | $\Delta N_{k}$ |  |  |  | $v_{k}$ |
| 1: $A D$ | $110^{\circ} 15^{\prime} 17^{\prime \prime}$ | 2401.609 | 2253.1002 | -831.4235 | 2113.7748 | 287.8342 | -780.0106 | 0.057 |
| 2: $D E$ | $68^{\circ} 34^{\prime} 12^{\prime \prime}$ | 1032.340 | 960.9688 | 377.1801 | 894.5319 | 137.8081 | 351.1036 | -0.168 |
| 3: EF | $163^{\circ} 03^{\prime} 23^{\prime \prime}$ | 559.022 | 162.9160 | -534.7560 | 47.4788 | 511.5433 | -155.8442 | 0.129 |
| 4: FB | $113^{\circ} 49^{\prime} 38^{\prime \prime}$ | 1564.683 | 1431.3217 | -632.1006 | 1309.3270 | 255.3560 | -578.2253 | 0.064 |
|  | sums | 5557.6540 | 4808.3067 | -1621.1000 | 4365.1124 | 1192.5416 | -1162.9764 |  |

$S_{E}=\sum_{k=1}^{n} \Delta E_{k}=4808.3067, S_{N}=\sum_{k=1}^{n} \Delta N_{k}=-1621.1000$
$a=\sum_{k=1}^{n} \frac{\left(\Delta E_{k}\right)^{2}}{d_{k}}=4365.1124, b=\sum_{k=1}^{n} \frac{\left(\Delta N_{k}\right)^{2}}{d_{k}}=1192.5416$ and $c=\sum_{k=1}^{n} \frac{\Delta E_{k} \Delta N_{k}}{d_{k}}=-1162.9764$
$D_{E}=E_{\text {END }}-E_{\text {START }}=6843.085-2034.785=4808.300$
$D_{N}=N_{\text {END }}-N_{\text {START }}=7154.700-8776.030=-1621.330$
$D_{E}-S_{E}=-0.0067, D_{N}-S_{N}=-0.2300$

The multipliers are: $\quad k_{1}=\frac{b\left(D_{E}-S_{E}\right)-c\left(D_{N}-S_{N}\right)}{a b-c^{2}}=-7.1501 e-05$

$$
k_{2}=\frac{a\left(D_{N}-S_{N}\right)-c\left(D_{E}-S_{E}\right)}{a b-c^{2}}=-2.6259 e-04
$$

The residuals are: $\quad v_{k}=k_{1} \Delta E_{k}+k_{2} \Delta N_{k}$
The adjusted traverse (nearest mm) is

| Line | Bearing | Distance |
| :---: | :---: | :---: |
| $k$ | $\phi_{k}$ | $d_{k}$ |
| $1: A B$ | $110^{\circ} 15^{\prime} 17^{\prime \prime}$ | 2401.666 |
| $2: B C$ | $68^{\circ} 34^{\prime} 12^{\prime \prime}$ | 1032.172 |
| $3: C D$ | $163^{\circ} 03^{\prime} 23^{\prime \prime}$ | 559.151 |
| $4: D A$ | $113^{\circ} 49^{\prime} 38^{\prime \prime}$ | 1564.747 |

Using the program: press XEQ A001 (or XEQ $\mathbf{A}$ ENTER )
Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry
When all sides have been keyed in, enter 0 at the prompt B ? and press R/S; and 0 at the prompt D ? and press R/S (or simply press R/S at both prompts).

At the prompt X? enter 4808.300 and press R/S ( $D_{E}=4808.300$ )
At the prompt Y? enter -1621.330 and press $\mathrm{R} / \mathrm{S}\left(D_{N}=-1621.330\right)$
At the prompt F? enter 2 and press R/S.
The calculator will then display the adjusted bearing at $\mathrm{B}=$. Press $\mathrm{R} / \mathrm{S}$ and the adjusted distance will be displayed at $\mathrm{D}=$. Repeat pressing of $\mathrm{R} / \mathrm{S}$ will display adjusted bearings and distances.
After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $\mathrm{A}=$ (The area $=-357,597.8300 \mathrm{~m}^{2}$ but is meaningless since this is not a closed polygon)

| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A001 | LBL A |  |  |  |  |
| A002 | CLVARS |  | START NEW ADJUSTMENT | ADJUSTMENT |  |
| A003 | CLE |  |  |  |  |
| A004 | -1 |  |  |  |  |
| A005 | STO I |  |  |  |  |
| A006 | 2 |  | START NEW LINE OF FIGURE |  |  |
| A007 | STO+I | Increment | indirect storage registers |  |  |
| A008 | STO+J |  |  |  |  |
| A009 | 0 |  |  |  |  |
| A010 | STO B |  |  |  |  |
| A011 | STO D |  |  |  |  |
| A012 | INPUT B | Enter Bearing (D.MMSS) |  |  |  |
| A013 | HMS $\rightarrow$ |  |  |  |  |
| A014 | STO B |  |  |  |  |
| A015 | INPUT D | Enter Distance $d_{k}$ |  |  |  |
| A016 | STO+L | Accumulate distances |  |  |  |
| A017 | RCL B | Brg | Dist |  |  |
| A018 | + | Brg+Dist |  |  |  |
| A019 | $x=0$ ? |  |  |  |  |
| A020 | GT0 A044 | Yes! End of Data; GO TO adjustment |  |  |  |
| A021 | RCL B | Brg |  |  |  |
| A022 | RCL D | Dist | Brg |  |  |
| A023 | XEQ Z015 | $\Delta N_{k}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A024 | $\Sigma+$ | n | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A025 | LASTX | $\Delta N_{k}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A026 | STO ( J ) |  |  |  |  |
| A027 | $x<>y$ | $\Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta \mathrm{N}_{\mathrm{k}}$ |  |  |
| A028 | STO ( I ) |  |  |  |  |
| A029 | $\times$ | $\Delta \mathrm{E}_{\mathrm{k}} \Delta \mathrm{N}_{\mathrm{k}}$ |  |  |  |
| A030 | RCL D | $\mathrm{d}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}} \Delta \mathrm{N}_{\mathrm{k}}$ |  |  |
| A031 | $\div$ | $\Delta \mathrm{E}_{\mathrm{k}} \Delta \mathrm{N}_{\mathrm{k}} / \mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A032 | ST0+S |  |  |  |  |
| A033 | RCL ( I ) | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |  |
| A034 | $x^{2}$ | $\left(\Delta \mathrm{E}_{\mathrm{k}}\right)^{2}$ |  |  |  |
| A035 | RCL D | $\mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A036 | $\div$ | $\left(\Delta \mathrm{E}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A037 | ST0+R |  |  |  |  |
| A038 | RCL ( J ) | $\Delta N_{k}$ |  |  |  |
| A039 | $x^{2}$ | $\left(\Delta N_{\mathrm{k}}\right)^{2}$ |  |  |  |
| A040 | RCL D | $\mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A041 | $\div$ | $\left(\Delta N_{k}\right)^{2} / d_{k}$ |  |  |  |


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A042 | STO+V |  |  |  |  |
| A043 | GT0 A006 | GO FOR next line |  |  |  |
| A044 | INPUT X | Enter $D_{E}$ |  |  |  |
| A045 | INPUT Y | Enter $D_{N}$ |  |  |  |
| A046 | INPUT F | Enter Flag (Bowditch = 1; Crandall = 2) |  |  |  |
| A047 | RCL F | Flag |  |  |  |
| A048 | 1 | 1 | Flag |  |  |
| A049 | $x=y$ ? |  |  |  |  |
| A050 | GT0 A056 | Yes! GO TO Bowditch adjustment |  |  |  |
| A051 | RCL F | Flag |  |  |  |
| A052 | 2 | 1 | Flag |  |  |
| A053 | $x=y$ ? |  |  |  |  |
| A054 | GT0 A079 | Yes! GO TO Crandall adjustment |  |  |  |
| A055 | GT0 A046 |  |  |  |  |
| A056 | XEQ A128 | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ | BOWDITCH ADJUSTMENT |  |
| A057 | RCL L | L | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |
| A058 | $\div$ | $\left(D_{N}-S_{N}\right) / L$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |  |
| A059 | STO Y |  |  |  |  |
| A060 | $x<>y$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ | $\left(D_{N}-S_{N}\right) / L$ |  |  |
| A061 | RCL L | L | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ | $\left(D_{N}-S_{N}\right) / L$ |  |
| A062 | $\div$ | $\left(D_{E}-S_{E}\right) / L$ | $\left(D_{N}-S_{N}\right) / L$ |  |  |
| A063 | ST0 X |  |  |  |  |
| A064 | XEQ A135 | Set registers C,I, J, A |  |  |  |
| A065 | XEQ A144 | Increment counters for next line of adjusted figure |  |  |  |
| A066 | XEQ A151 | Get UNADJUSTED Bearing and Distance |  |  |  |
| A067 | RCL D | $\mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A068 | RCL X | $\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right) / \mathrm{L}$ | $\mathrm{d}_{\mathrm{k}}$ |  |  |
| A069 | $\times$ | $\mathrm{dE}_{\mathrm{k}}=\mathrm{d}_{\mathrm{k}}\left[\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right) / \mathrm{L}\right]=$ correction to $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |  |
| A070 | ST0+( I ) |  |  |  |  |
| A071 | RCL D | $\mathrm{d}_{\mathrm{k}}$ |  |  |  |
| A072 | RCL Y | $\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right) / \mathrm{L}$ | $\mathrm{d}_{\mathrm{k}}$ |  |  |
| A073 | $\times$ | $\mathrm{dN}_{\mathrm{k}}=\mathrm{d}_{\mathrm{k}}\left[\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right) / \mathrm{L}\right]=$ correction to $\Delta \mathrm{N}_{\mathrm{k}}$ |  |  |  |
| A074 | ST0+( J ) |  |  |  |  |
| A075 | XEQ A162 | Compute Area contribution of ADJ. line |  |  |  |
| A076 | XEQ A151 | Get ADJUSTED Bearing and Distance |  |  |  |
| A077 | XEQ A177 | View Adjusted Bearing and Distance |  |  |  |
| A078 | GT0 A065 | GO FOR next line of figure |  |  |  |
| A079 | RCL R | $\mathrm{a}=\Sigma\left[\left(\Delta \mathrm{E}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}\right]$ |  | CRANDALL ADJUSTMENT |  |
| A080 | RCL V | $\mathrm{b}=\Sigma\left[\left(\Delta \mathrm{N}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}\right]$ |  | $\mathrm{a}=\Sigma\left[\left(\Delta \mathrm{E}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}\right]$ |  |
| A081 | $\times$ | ab |  |  |  |
| A082 | RCL S | $\left.\mathrm{c}=\Sigma\left[\Delta \mathrm{E}_{\mathrm{k}} \Delta \mathrm{N}_{\mathrm{k}}\right) / \mathrm{d}_{\mathrm{k}}\right]$ |  | ab |  |
| A083 | STO U | C | ab |  |  |


| LINE | STEP | X | $Y$ | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A084 | $x^{2}$ | $\mathrm{c}^{2}$ | ab |  |  |
| A085 | - | $a b-c^{2}$ |  |  |  |
| A086 | STO T |  |  |  |  |
| A087 | XEQ A128 | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |  |
| A088 | STO $\times$ S |  |  |  |  |
| A089 | STO $\times$ R |  |  |  |  |
| A090 | $x<>y$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ |  |  |
| A091 | STO $\times$ V |  |  |  |  |
| A092 | STO $\times$ U |  |  |  |  |
| A093 | RCL R | $a\left(D_{N}-S_{N}\right)$ |  |  |  |
| A094 | RCL U | $c\left(D_{E}-S_{E}\right)$ | $\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)$ |  |  |
| A095 | - | $\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)$ | ( $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ ) |  |  |
| A096 | RCL T | $a b-c^{2}$ | $\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ ) |  |
| A097 | $\div$ | $\mathrm{k}_{2}=\left[\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}\right.\right.$ | ) $-\mathrm{C}\left(\mathrm{D}_{\mathrm{E}}-S^{\text {c }}\right.$ | $/\left(a b-c^{2}\right)$ |  |
| A098 | STO W |  |  |  |  |
| A099 | RCL V | $\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)$ |  |  |  |
| A100 | RCL S | $c\left(D_{N}-S_{N}\right)$ | $\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)$ |  |  |
| A101 | - | $\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)$ | $\left(D_{N}-S_{N}\right)$ |  |  |
| A102 | RCT T | $a b-c^{2}$ | $b\left(D_{E}-S_{E}\right)$ | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ ) |  |
| A103 | $\div$ | $\mathrm{k}_{1}=\left[\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}\right.\right.$ | ) $-\mathrm{C}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}\right.$ | $/\left(a b-c^{2}\right)$ |  |
| A104 | STO T |  |  |  |  |
| A105 | XEQ A135 | Set regi | ers C, I, |  |  |
| A106 | XEQ A144 | Incremen adjusted | counters igure | or next |  |
| A107 | XEQ A151 | Get UNAD | STED Bea | gr and D |  |
| A108 | RCL ( I ) | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |  |
| A109 | RCL T | $\mathrm{k}_{1}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A110 | $\times$ | $\mathrm{k}_{1} \Delta \mathrm{E}_{\mathrm{k}}$ |  |  |  |
| A111 | RCL ( J ) | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\mathrm{k}_{1} \Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A112 | RCL W | $\mathrm{k}_{2}$ | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\mathrm{k}_{1} \Delta \mathrm{E}_{\mathrm{k}}$ |  |
| A113 | $\times$ | $\mathrm{k}_{2} \Delta \mathrm{~N}_{\mathrm{k}}$ | $\mathrm{k}_{1} \Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A114 | + | $\mathrm{V}_{\mathrm{k}}=\mathrm{k}_{1} \Delta \mathrm{E}_{\mathrm{k}}+\mathrm{k}$ | $\Delta N_{\mathrm{k}}$ |  |  |
| A115 | STO+D |  |  |  |  |
| A116 | RCL B |  |  |  |  |
| A117 | HMS $\rightarrow$ | Brg |  |  |  |
| A118 | RCL D | Dist | Brg |  |  |
| A119 | XEQ Z015 | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A120 | STO ( J ) |  |  |  |  |
| A121 | $x<>y$ | $\Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta N_{k}$ |  |  |
| A122 | STO ( I ) |  |  |  |  |
| A123 | XEQ A162 | Compute Area contribution of ADJ. line |  |  |  |
| A124 | XEQ A177 | View Adjusted Bearing and Distance |  |  |  |
| A125 | GT0 A106 | GO FOR next line of figure |  |  |  |
| A126 | VIEW A | Area |  |  |  |


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A127 | GT0 A002 | GO FOR new | figure to | adjust |  |
| A128 | RCL X | $\mathrm{D}_{\mathrm{E}}$ |  |  |  |
| A129 | $\Sigma y$ | $\mathrm{S}_{\mathrm{E}}=\Sigma \Delta \mathrm{E}_{\mathrm{k}}$ | $\mathrm{D}_{\mathrm{E}}$ |  |  |
| A130 | - | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |  |  |
| A131 | RCL Y | $\mathrm{D}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |  |
| A132 | $\Sigma x$ | $\mathrm{S}_{\mathrm{N}}=\Sigma \Delta \mathrm{N}_{\mathrm{k}}$ | $\mathrm{D}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |
| A133 | - | $\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}$ | $\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}$ |  |  |
| A134 | RTN |  |  |  |  |
| A135 | n | n | SET REGISTERS C, I, J, A |  |  |
| A136 | STO C | count = n |  |  |  |
| A137 | CLE |  |  |  |  |
| A138 | -1 | -1 |  |  |  |
| A139 | STO I |  |  |  |  |
| A140 | 0 | 0 |  |  |  |
| A141 | STO J |  |  |  |  |
| A142 | STO A |  |  |  |  |
| A143 | RTN |  |  |  |  |
| A144 | 2 | 2 | INCREMENT REGISTERS I, J |  |  |
| A145 | STO+I | Increment indirect storage reg. for $\Delta E$ |  |  |  |
| A146 | ST0+J | Increment indirect storage reg. for $\Delta N$ | indirect storage reg. for $\Delta N$ |  |  |
| A147 | RCL C | count |  |  |  |
| A148 | $x=0$ ? |  |  |  |  |
| A149 | GT0 A126 | Yes! GO FOR Area of adjusted figure |  |  |  |
| A150 | RTN |  |  |  |  |
| A151 | 360 | 360 | BEARING \& | DISTANCE S | UBROUTINE |
| A152 | RCL ( I ) | $\Delta \mathrm{E}_{\mathrm{k}}$ | 360 |  |  |
| A153 | RCL ( J ) | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ | 360 |  |
| A154 | XEQ Z002 | $\mathrm{d}_{\mathrm{k}}$ | $\mathrm{Br} \mathrm{g}_{\mathrm{k}}$ | 360 |  |
| A155 | STO D |  |  |  |  |
| A156 | $\mathrm{R} \downarrow$ | Brgk | 360 |  |  |
| A157 | $x<0$ ? |  |  |  |  |
| A158 | + |  |  |  |  |
| A159 | $\rightarrow$ HMS | Brg(D.MMSS) |  |  |  |
| A160 | STO B |  |  |  |  |
| A161 | RTN |  |  |  |  |
| A162 | RCL ( I ) | $\Delta \mathrm{E}_{\mathrm{k}}$ | AREA SUBROUTINE |  |  |
| A163 | RCL ( J ) | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A164 | $\Sigma+$ | n | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A165 | R $\downarrow$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |  |
| A166 | LASTX | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A167 | $\Sigma y$ | $\Sigma \Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |
| A168 | $\times$ | $\Delta \mathrm{N}_{\mathrm{k}} \Sigma \Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A169 | $x<>y$ | $\Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta \mathrm{N}_{\mathrm{k}} \Sigma \Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A170 | $\Sigma x$ | $\Sigma \Delta N_{\mathrm{k}}$ | $\Delta \mathrm{E}_{\mathrm{k}}$ | $\Delta \mathrm{N}_{\mathrm{k}} \Sigma \Delta \mathrm{E}_{\mathrm{k}}$ |  |


| LINE | STEP | X | Y | Z | T |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A171 | $\times$ | $\Delta \mathrm{E}_{\mathrm{k}} \Sigma \Delta \mathrm{N}_{\mathrm{k}}$ | $\Delta \mathrm{N}_{\mathrm{k}} \Sigma \Delta \mathrm{E}_{\mathrm{k}}$ |  |  |
| A172 | - | $\Delta \mathrm{N}_{\mathrm{k}} \Sigma \Delta \mathrm{E}_{\mathrm{k}}-\Delta \mathrm{E}_{\mathrm{k}} \Sigma \Delta \mathrm{N}_{\mathrm{k}}$ |  |  |  |
| A173 | 2 |  |  |  |  |
| A174 | $\div$ | Area component |  |  |  |
| A175 | STO+A | Accumulate area |  |  |  |
| A176 | RTN |  |  |  |  |
| A177 | VIEW B | (Adjusted) Bearing (D.MMSS) |  |  |  |
| A178 | VIEW D | Adjusted Distance |  |  |  |
| A179 | 1 | 1 |  |  |  |
| A180 | STO-C | Decreme | count |  |  |
| A181 | RTN |  |  |  |  |

## STORAGE REGISTERS

| A | Area |  |
| :---: | :---: | :---: |
| B | Bearing |  |
| C | count $=$ counter for lines of figure | $0 \leq$ count $\leq$ |
| D | Distance $\mathrm{d}_{\mathrm{k}}$; $\mathrm{d}_{\mathrm{k}}+\mathrm{V}_{\mathrm{k}}$ |  |
| 1 | Indirect storage register for $\Delta \mathrm{E}$ |  |
| J | Indirect storage register for $\Delta N$ |  |
| L | Cumulative distance $\mathrm{L}=\Sigma \mathrm{d}_{\mathrm{k}}$ |  |
| R | $\mathrm{a}=\Sigma\left[\left(\Delta \mathrm{E}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}\right]$; $\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)$ |  |
| S | $\left.\mathrm{c}=\Sigma\left[\Delta \mathrm{E}_{\mathrm{k}} \Delta \mathrm{N}_{\mathrm{k}}\right) / \mathrm{d}_{\mathrm{k}}\right]$; $\mathrm{c}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)$ |  |
| T | $\mathrm{ab}-\mathrm{c}^{2} ; \mathrm{k}_{1}=\left[\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)-\mathrm{c}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{N}\right)\right] /\left(\mathrm{ab}-\mathrm{c}^{2}\right)$ |  |
| U | c ; $\mathrm{c}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)$ |  |
| V | $\mathrm{b}=\Sigma\left[\left(\Delta \mathrm{N}_{\mathrm{k}}\right)^{2} / \mathrm{d}_{\mathrm{k}}\right]$; $\mathrm{b}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)$ |  |
| W | $\mathrm{k}_{2}=\left[\mathrm{a}\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right)-\mathrm{c}\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right)\right] /\left(\mathrm{ab}-\mathrm{c}^{2}\right)$ |  |
| X | $\mathrm{D}_{\mathrm{E}}=\mathrm{E}_{\text {END }}-\mathrm{E}_{\text {START }}$; $\left(\mathrm{D}_{\mathrm{E}}-\mathrm{S}_{\mathrm{E}}\right) / \mathrm{L}$ |  |
| Y | $\mathrm{D}_{\mathrm{N}}=\mathrm{N}_{\text {END }}-\mathrm{N}_{\text {START }} ; ~\left(\mathrm{D}_{\mathrm{N}}-\mathrm{S}_{\mathrm{N}}\right) / \mathrm{L}$ |  |

## PROGRAM LENGTH AND CHECKSUM

$L N=558 ; C K=68 A 4$


## PROGRAM NOTES



The calculator must contain LBL Z which contains the Polar to Rectangular routines.

XEQ Z002 on line A154 is the Rectangular $\rightarrow$ Polar conversion XEQ Z015 on lines A023,A119 is the Polar $\rightarrow$ Rectangular conversion.


[^0]:    ${ }^{1}$ A mathematical adjustment of chain and compass surveys developed by the American mathematician and astronomer Nathaniel Bowditch (1773-1838). This adjustment affects both bearings and distances.
    ${ }^{2}$ A mathematical 'least squares' adjustment of traverse distances only that assumes that observed bearings 'close' perfectly. Developed in 1906 by Charles L. Crandall, Professor of Railroad Engineering and Geodesy, Cornell University, New York.
    Theory and examples of Bowditch's and Crandall's adjustments can be found in Notes on Least Squares, Geospatial Science, RMIT University, Chapter 6, pp.6-15 - 6-26.

