HP 35s Surveying Programs



CLOSURE with Accuracy, Area and double-missing distance Coordinate RADIATIONS with rotation and scale Coordinate JOINS Radiations from OFFSETS RESECTION ADJUSTMENT – Bowditch and Crandall

Geospatial Science, RMIT University

HP35s SURVEYING PROGRAMS

- 1. The following programs have been collated for the use of students in the Surveying and Geospatial Science programs in the School of Mathematical and Geospatial Sciences, RMIT University. As always, it is the user's responsibility to ensure that the programs are installed correctly and then checked. Also, do not alter programs unless you are aware of what LABELS are being used or whether GTO and BRANCHING label addresses will be affected; because by doing so you may dramatically affect the way they work and hence obtain incorrect answers.
- 2. The following two programs under LABEL Z are critical and must be kept in your HP35s at all times. <u>Do not delete them!</u>
 - RECTANGULAR \rightarrow POLAR XEQ Z002
 - POLAR \rightarrow RECTANGULAR XEQ Z015

These programs are software replacements for the Polar \leftrightarrows Rectangular conversion functions that were present on the HP33s and HP32s calculators and have not been implemented on the HP35s.

3. The following are a 'suite' of surveying computation programs that will be useful in the field and office. Some (Closure, Radiations, Joins, Offsets) have a heritage extending back to HP desktop-computer programs from the 1970's written by Bodo Taube of Francis O'Halloran, Surveyors. And Bodo Taube's programs were (and are) models of efficiency. Others are more recent.

Each program has a set of User Instructions, with examples and relevant formula and HP35s Program Sheets listing the program steps (that you may key into your calculator), storage registers used and program notes.

- CLOSURE XEQ C001
- RADIATIONS XEQ R001
- JOINS XEQ J001
- OFFSETS XEQ 0001
- RESECTION XEQ S001
- ADJUSTMENT XEQ A001

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HP35s POLAR RECTANGULAR CONVERSIONS

The following programming code is a software replacement for the POLAR≒RECTANGULAR conversion functions that were present on the HP33s and HP32s calculators and have not been implemented on the HP35s.

This code was made available through *The Museum of HP Calculators* and appeared in HP Forum Archive 17 (22-Aug-2007)

http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv017.cgi?read=122519

RECTA	NGULAR→POLAR →RECTANGULAR	XEQ Z002 XEQ 3 0 2 XEQ Z015 XEQ 3 0 1 5
LINE	STEP	KEYSTROKES
Z001	LBL Z	XEQ 3
Z002	CF 10	
Z003	ABS	
Z004	CLx	
Z005	LASTX	
Z006	R♥	R↓
Z007	R♥	R↓
Z008	Eqn REGZ+i*REGT	EQN $R\downarrow$ 3 + i ×
Z009	ENTER	
Z010	R♥	R↓
Z011	R★	R↓
Z012	Eqn ARG(REGT)	
Z013	Eqn ABS(REGT)	
Z014	RTN	XEQ
Z015	CF 10	
Z016	ABS	┌── +/-
Z017	R♥	R↓
Z018	R♥	R↓



What do these pieces of code do?

RECTANGULAR+**POLAR**



POLAR→RECTANGULAR

Т			Т	
Ζ			Ζ	
Y	Bearing	XEQ Z015	Υ	East
X	Distance	XEQ 3 0 1 5	Х	North

The contents of registers Z and T remain unchanged for both conversions.

MISSING BEARING & DISTANCE OR DOUBLE MISSING DISTANCE (BEARING INTERSECTION) WITH AREA

PRESS XEQ C001 TO RUN PROGRAM

- Notes: 1. For *missing bearing and distance*, the missing line must be the last line in the closure.
 - 2. For *double missing distance*, the missing distances must be on the last two lines of the closure.
 - 3. Missing elements must be input as zero, i.e., if the bearing is unknown then enter 0 when requested and if the distance is unknown enter 0 when requested.
 - 4. Bearings of lines that are 0° 00' 00" must be entered as 360° 00' 00"



AREA ALGORITHM
$$\Delta \operatorname{Area}_{k} = -\frac{1}{2} \left\{ \Delta N_{k} \sum_{i=1}^{k} \Delta E_{i} - \Delta E_{k} \sum_{i=1}^{k} \Delta N_{i} \right\}$$

HP35s PROGRAM

EXAMPLES

- 1. Closure with: (i) misclose bearing and distance;
 - (ii) misclose east and north;
 - (iii) misclose accuracy; and
 - (iv) area



Figure ABCDEF is section of road 20 m wide that is being excised from an allotment of land.

Check that the dimensions are correct and determine the area.

Starting with the line AB and going clockwise around the figure, enter the bearing and distance of each side, remembering that the bearing of the last side FA should be entered as 360° 00'.

Enter 0 for the last bearing and 0 for the last side (you don't have to key anything in; just press R/S at the prompts) since the last side (the misclose) is unknown.

The calculator will display: F Press R/S The calculator will display: I	B = 136.09 D = 0.0021	24 (136° 09' 24") (the misclose beat (the misclose distance);	uring);
Press R/S	0.001.4		
The calculator will display:	0.0014 -0.0015	(east misclose) (north misclose); often shown as	<u> </u>
Press R/S			
The calculator will display: Press R/S	R = 502,2	288.7039 (this is the misclose accur	racy ratio 1:502289)
The calculator will display:	A = -9,92 (the negative	26.0706 (this is the area 9926 m ²) tive sign is due to entering the figure	re clockwise)
Press R/S The calculator will display:	B? 0.0000		

Ready for the next closure.

HP35s PROGRAM

EXAMPLES

2. Closure with: (i) double missing and distance; and (ii) area



Figure ABCDEF is section of road 20 m wide that is being excised from an allotment of land.

Compute the missing distances *CD* and *DE*, and the area.

Starting with the line EF and going clockwise around the figure, enter the bearing and distance of each 'known' side, remembering that the bearing of the side FA should be entered as 360° 00'.

Enter the bearing of the side CD and 0 for the distance (the 1st missing distance; you don't have to key anything in; just press R/S at the prompt). Enter the bearing of the side ED. The calculator will now solve for the two missing distances CD and DE.

The calculator will display:	D = 20.0907 (the 1st missing distance);
Press R/S	
The calculator will display:	D = 204.5581 (the 2nd missing distance);
Press R/S	
The calculator will display:	A = -9,926.6036 (this is the area 9926 m^2) (the negative sign is due to entering the figure clockwise)

Press R/S The calculator will display: B? 0.0000

Ready for the next closure.

NOTE: For double missing distance closures, the missing sides must be the last two sides. To achieve this, some figures may need re-casting. In such cases, the areas of re-cast figures may not be correct. See the following example

EXAMPLES

3. Closure with: (i) double missing and distance; and (ii) area



Figure ABCDEF is section of road 20 m wide that is being excised from an allotment of land.

Compute the missing distances *AB* and *CD*, and the area.

Re-cast the figure so that the last two sides contain the missing distances



Starting with the line DE and going clockwise around the re-cast figure, enter the bearing and distance of each 'known' side, remembering that the bearing of the side FA should be entered as 360° 00'.

Enter the bearing of the side B'C and 0 for the distance (the 1st missing distance; you don't have to key anything in; just press R/S at the prompt).

Enter the bearing of the side *CD*. The calculator will now solve for the two missing distances B'C and *CD*.

The calculator will display: D = 292.7520 (the 1st missing distance); Press R/S The calculator will display: D = 20.0916 (the 2nd missing distance); Press R/S The calculator will display: A = 18,126.6222 (this is complete rubbish since the lines in the re-cast figure cross)

AREA ALGORITHM

The algorithm for computing the area of a polygon can be derived by considering Figure A1, where the area is the sum of the trapeziums *bBCc*, *cCDd* and *dDEe* less the triangles *bBA* and *AEe*.

The area can be expressed as

$$2A = \left[(x_2 - x_1) + (x_3 - x_1) \right] \left[(y_2 - y_3) \right] \\ + \left[(x_3 - x_1) + (x_4 - x_1) \right] \left[(y_3 - y_4) \right] \\ + \left[(x_4 - x_1) + (x_5 - x_1) \right] \left[(y_4 - y_5) \right] \quad (A1) \\ - (x_2 - x_1) (y_2 - y_1) \\ - (x_5 - x_1) (y_1 - y_5)$$

Expanding (A1) then cancelling and rearranging terms gives

$$2A = x_{1}(y_{5} - y_{2}) +x_{2}(y_{1} - y_{3}) +x_{3}(y_{2} - y_{4}) +x_{4}(y_{3} - y_{5}) +x_{5}(y_{4} - y_{1})$$

$$y = e \\ E_{x_{5}}y_{5} \\ x \\ Figure A1$$

which can be expressed as $2A = \sum_{k=1}^{n} \{x_k (y_{k-1} - y_{k+1})\}$ (A2)

In Figure A2, the coordinate origin is shifted to A where $x'_1 = y'_1 = 0$ and the area, using (A2), is

$$2A = y_2'x_3' + y_3'x_4' - y_3'x_2' + y_4'x_5' - y_4'x_3' - y_5'x_4'$$

Considering each side of the polygon to have components Δx_k , Δy_k for k = 1 to 5, (A3) can be written as

$$2A = \Delta y_1 (\Delta x_1 + \Delta x_2) + (\Delta y_1 + \Delta y_2) (\Delta x_1 + \Delta x_2 + \Delta x_3) - (\Delta y_1 + \Delta y_2) (\Delta x_1) + (\Delta y_1 + \Delta y_2 + \Delta y_3) (\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4) - (\Delta y_1 + \Delta y_2 + \Delta y_3) (\Delta x_1 + \Delta x_2) - (\Delta y_1 + \Delta y_2 + \Delta y_3 + \Delta y_4) (\Delta x_1 + \Delta x_2 + \Delta x_3)$$





Expanding and gathering terms gives

$$2A = \Delta y_1 (3\Delta x_1 + 3\Delta x_2 + 2\Delta x_3 + \Delta x_4) - \Delta y_1 (3\Delta x_1 + 2\Delta x_2 + \Delta x_3) + \Delta y_2 (2\Delta x_1 + 2\Delta x_2 + 2\Delta x_3 + \Delta x_4) - \Delta y_2 (3\Delta x_1 + 2\Delta x_2 + \Delta x_3) + \Delta y_3 (\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4) - \Delta y_3 (2\Delta x_1 + 2\Delta x_2 + \Delta x_3) - \Delta y_4 (\Delta x_1 + \Delta x_2 + \Delta x_3)$$

and cancelling terms and re-ordering gives

$$2A = \Delta y_{1} \left(0 + \Delta x_{2} + \Delta x_{3} + \Delta x_{4} \right) + \Delta y_{2} \left(-\Delta x_{1} + 0 + \Delta x_{3} + \Delta x_{4} \right) + \Delta y_{3} \left(-\Delta x_{1} - \Delta x_{2} + 0 + \Delta x_{4} \right) + \Delta y_{4} \left(-\Delta x_{1} - \Delta x_{2} - \Delta x_{3} + 0 \right)$$
(A4)

This equation for the area can also be expressed as a matrix equation

$$2A = \begin{bmatrix} \Delta y_1 & \Delta y_2 & \Delta y_3 & \Delta y_4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix}$$
(A5)

By studying the form of equations (A4) and (A5), the following algorithm for calculating the k = n-1 area components A_k for a polygon of *n* sides may be deduced as

$$A_{k} = \frac{1}{2} \left\{ \Delta x_{k} \sum_{i=1}^{k} \Delta y_{i} - \Delta y_{k} \sum_{i=1}^{k} \Delta x_{i} \right\} \text{ where } k = 1, 2, 3, \dots n-1$$
 (A6)

Equation (A6) is an efficient way to accumulate the area of a polygon given the coordinate components of the sides. By studying the algorithm, it can be seen that $A_1 = A_n = 0$ and hence the area of a polygon is accumulated without having to deal with the last side. This makes it a very useful area algorithm for simple closure programs where the last side is often the missing side in the polygon. In addition, it can be seen that each area component A_k is a triangle with one vertex at the starting point and the line k, with components Δx_k , Δy_k , the opposite side.

Rearranging equation (A6) and expressing the components of lines as ΔE and ΔN where *E* and *N* are east and north respectively gives the area algorithm used in the HP35s Closure Program

$$A_{k} = -\frac{1}{2} \left\{ \Delta N_{k} \sum_{i=1}^{k} \Delta E_{i} - \Delta E_{k} \sum_{i=1}^{k} \Delta N_{i} \right\} \text{ where } k = 1, 2, 3, \dots n-1$$
 (A7)

HP35s PROGRAM SHEET CLOSURE PROGRAM

LINE	STEP	X	Y	Z	Т
C001	LBL C				
C002	CLVARS		START NEW	CLOSURE	
C003	$\texttt{CL}\Sigma$				
C004	0		NEW LINE C	F CLOSURE	
C005	STO B				
C006	STO D				
C007	INPUT B	Enter Bear	ring	ſ	ſ
C008	HMS→				
C009	STO B				
C010	STO C				
C011	INPUT D	Enter Dist	cance		
C012	STO+R	accumulate	e distances	ſ	[
C013	RCL B	Brg	Dist		
C014	+	Brg+Dist			
C015	x = 0 ?	test to se	ee if both	Brg & Dist	are zero
C016	GTO C068	go for mis	sing beari	ng & distan	nce
C017	RCL D				
C018	x = 0 ?	test to se	ee if Dist	is zero	
C019	GTO C042	go for dou	ıble missin	g distance	
C020	XEQ C022	compute ar	rea contrib	ution for .	line
C021	GTO C004	go for nex	t line of	closure	
C022	RCL B	Brg	AREA SUBRO	UTINE	
C023	RCL D	Dist	Brg		
C024	XEQ Z015	$\Delta \mathrm{N}$	$\Delta { ext{E}}$		
C025	Σ +	n	$\Delta { ext{E}}$		
C026	R↓	ΔE			
C027	LASTX	Δ N	$\Delta { ext{E}}$		
C028	Σy	Σ (Δ E)	Δ N	ΔE	
C029	×	$\Delta { m N}$ (Σ ($\Delta { m E}$))	ΔE		
C030	<i>x</i> <> <i>y</i>	ΔE	$\Delta { m N}$ (Σ ($\Delta { m E}$))		
C031	Σx	Σ (Δ N)	$\Delta { ext{E}}$	$\Delta { m N}$ (Σ ($\Delta { m E}$))	
C032	×	$\Delta { m E}$ (Σ ($\Delta { m N}$))	$\Delta { m N}$ (Σ ($\Delta { m E}$))		
C033	-	$\Delta \mathrm{N}$ (Σ ($\Delta \mathrm{E}$)) -	$-\Delta E (\Sigma (\Delta N))$		
C034	2				
C035	÷	area compo	onent		
C036	STO+A	accumulate	e area		
C037	RTN				
C038	Σy	$\Sigma (\Delta E)$	BRG & DIST	SUBROUTIN	E
C039	Σx	$\Sigma (\Delta N)$	$\Sigma (\Delta E)$		
C040	XEQ Z002	Dist	Brg		
C041	RTN				

HP35s PROGRAM SHEET CLOSURE PROGRAM

LINE	STEP	X	Y	Z	Т
C042	0		DOUBLE MIS	SING DISTA	NCE
C043	STO B				
C044	INPUT B	Enter 2nd	Bearing B_n^1		
C045	HMS→				
C046	STO B	B_n^1			
C047	RCL C	B_{n-1}^n	B_n^1		
C048	-	$\pm(180-\gamma)$			
C049	SIN	$\pm \sin \gamma$			
C050	XEQ C038	С	B_1^{n-1}	$\pm \sin \gamma$	
C051	x <> y	B_1^{n-1}	с	$\pm \sin \gamma$	
C052	RCL B	B_n^1	B_1^{n-1}	с	$\pm \sin \gamma$
C053	-	$\pm(180-\alpha)$	с	$\pm \sin \gamma$	
C054	SIN	$\pm \sin \alpha$	с	$\pm \sin \gamma$	
C055	×	$\pm c \sin \alpha$	$\pm \sin \gamma$		
C056	x <> y	$\pm \sin \gamma$	$\pm c \sin \alpha$		
C057	÷	±a (1st mi	ssing dist	ance)	
C058	STO D	$\pm a$			
C059	RCL C	B_{n-1}^n	$\pm a$		
C060	STO B				
C061	XEQ C022	compute ar	rea contrib	ution for	line
C062	VIEW D	lst Missir	ng Distance	P	P
C063	XEQ C038				
C064	STO D				
C065	VIEW D	2nd Missir	ng Distance		
C066	VIEW A	Area	1	1	1
C067	GTO COO2				
C068	XEQ C038	Dist	Brg	MISSING BE	RG & DIST
C069	STO D				
C070	STO÷R				
C071	x <> y				
C072	180				
C073	+				
C074	→HMS	Brg			
C075	STO B				
C076	VIEW B	Missing Be	earing	•	•
C077	VIEW D	Missing Di	stance		
C078	Σy	Σ (Δ E)			
C079	+/-	$-\Sigma~(~\Delta { m E}~)$			
C080	Σx	Σ (Δ N)	$-\Sigma (\overline{\Delta E})$		
C081	+ / -	$-\Sigma$ (Δ N)	$-\Sigma$ (Δ E)		
C082	STOP	N miscl.	E miscl.		

SHEET 2 OF 3 SHEETS

LINE	STEP	X	Y	Z	Т		
C083	VIEW R	Misclose Accuracy 1:x					
C084	VIEW A	Area					
C085	GTO C002						

STORAGE REGISTERS

Α	Area
В	Bearing
С	Bearing
D	Distance
R	Cumulative distance; closure accuracy

PROGRAM LENGTH AND CHECKSUM

LN = 261; CK = D83C

★ Length & Checksum:	► 2	; <	ENTER	(Hold)
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PROGRAM NOTES

Lines	C022	to	C037	is an area subroutine that also
				accumulates the east and north
				components of lines
Lines	C038	to	C041	is a subroutine to calculate a bearing
				and distance from east and north
				components of a line
Lines	C042	to	C067	is the double missing distance part of
				the closure program
Lines	C067	to	C085	is the missing line part of the
				closure program.

The calculator must contain LBL Z which contains the Polar to Rectangular routines

USER INSTRUCTIONS COORDINATE RADIATIONS PROGRAM

1. To start program press **XEQ R001**

2.	Display	E?	Enter:	East coordinate of traverse point; then press R/S
		0.0000		
3.	Display	N? 0.0000	Enter:	North coordinate of traverse point; then press R/S
4.	Display	R? 0.0000	Enter:	Rotation (\pm D.MMSS); then press R/S
5	Dieplay	S 2	Enter	Scale Eactor: then press R/S
5.	Display	1.0000	[If no sc	scale factor to be applied, press R/S and scale factor = 1]
6.	Display	B? 0.0000	Enter:	Radiation Bearing (D.MMSS); then press R/S
7.	Display	D? 0.0000	Enter: [If next	Radiation Distance; then press R/S Instrument Point, enter <u>distance with a negative sign</u> .]

7A If Rotation and Scale not 0° and 1; new bearing and distance displayed at successive R/S

8. East and North coordinate displayed at successive R/S. GoTo step 6.

In the example traverse below, with rotation = 0° and scale = 1, start at *A*, compute the coordinates of *A1* and *A2*; jump to *B*, compute coordinates of *B1* and *B2*; then to *C* and the coordinates of *C1*, *C2* and *C3*. The values in parentheses are for rotation = $+2^{\circ}$ 18' 35" and scale factor = 1.002515. (Distances and coordinates are rounded to nearest mm.)



HP35s PROGRAM SHEET RADIATIONS PROGRAM

LINE	STEP	X	Y	Z	Т			
R001	LBL R							
R002	CLVARS		START RADI	ATION PROG	RAM			
R003	CLΣ							
R004	INPUT E	Enter East	coordinat	e of Instru	ument Pt.			
R005	INPUT N	Enter Nort	ch coordina	te				
R006	CF 2	Clear Flag	Clear Flag 2					
R007	INPUT R	Enter ±Rot	Enter ±Rotation (D.MMSS)					
R008	HMS→							
R009	STO R	±Rotation						
R010	x = 0 ?							
R011	999							
R012	STO T							
R013	1							
R014	STO S							
R015	INPUT S	Enter Scal	le factor	·				
R016	STO×T							
R017	RCL T	rot×scale						
R018	999	999	rot×scale					
R019	x = y?							
R020	SF 2	Yes! Set H	Flag 2, no	rotation or	r scale			
R021	RCL E	Е						
R022	RCL N	Ν	Е	COORDS OF	I.P.			
R023	Σ +	n	Е					
R024	CF 1		RADIATION	TO NEW POI	NT			
R025	0							
R026	STO B							
R027	STO D							
R028	INPUT B	Enter Bear	cing (D.MMS	S)				
R029	INPUT D	Enter Dist	cance					
R030	<i>x</i> < 0 ?	±Dist	Test for n	legative di	stance			
R031	SF 1	Yes!, Set	Flag 1, ra	diation to	new I.P.			
R032	ABS	Dist						
R033	STO D							
R034	FS? 2	Test for r	otation an	d scale				
R035	GTO R046	No rotatio	on or scale					
R036	RCL B	Bearing						
R037	HMS→							
R038	RCL R	Rotation	Bearing					
R039	+	Rotated Be	earing					
R040	→HMS							
R041	STO B							
R042	RCL S	Scale						
R043	STO×D							

SHEET 1 OF 3 SHEETS

HP35s PROGRAM SHEET

LINE	STEP	X	Y	Z	Т
R044	VIEW B	Rotated Be	earing (D.M	MSS)	
RO45	VIEW D	Scaled Dis	stance		
R046	RCL B				
R047	HMS→	Brg			
R048	RCL D	Dist	Brg		
R049	XEQ Z015	$\Delta \mathrm{N}$	ΔE		
R050	STO+N	Δ N	ΔE		
R051	FS? 1	Test for r	new Instrum	ent Point	
R052	Σ +	n	ΔE	Yes! new I	.P.
R053	<i>x</i> <> <i>y</i>	ΔE	n		
R054	STO+E				
R055	VIEW E	East			
R056	VIEW N	North			
R057	Σx	North coor	d of Instr	ument Point	
R058	STO N				
R059	Σy	East coord	l of Instru	ment Point	
R060	STO E				
R061	GTO R024				

STORAGE REGISTERS

В	<pre>Bearing(D.MMSS); Bearing(Degree); Rotated Brg</pre>
D	Distance; Scaled Distance
E	East coordinate
Ν	North coordinate
R	Rotation (D.MMSS); Rotation (Degrees)
S	Scale factor
T	T=999 if Rotation = 0 degrees and Scale Factor = 1
•	T≠999 if any other Rotation and Scale Factor
Σx	North coordinate of Instrument Point
Σy	East coordinate of Instrument Point

PROGRAM LENGTH AND CHECKSUM

LN = 191; CK = 22C1

★ Length & Checksum: $\square \square \square 2$; $\square \square \square$ (Hold)

SHEET 2 OF 3 SHEETS

PROGRAM NOTES

Flag 1 is used to test to see if new point is to be next Instrument Point. Flag 2 is used to test to see if Rotation and Scale Factor is to be applied. Initialisation; storing coordinates of Lines R001 to R023 Instrument Point; storing Rotation and Scale. If Rotation = 0 degrees and Scale Factor = 1 then register T = 999. Any other value in T means that a Rotation and Scale Factor is assumed. Lines R024 to R033 Radiation Bearing and Distance to new point entered. If the distance is negative; Flag 1 is set and the new point will be the next Instrument Point. Lines R036 to R045 Rotated Bearing and Scaled Distance to new point displayed. Lines R046 to R060 Coordinates of new point computed. Ιf Flag 1 is set then the new point becomes the next Instrument Point.

The calculator must contain LBL Z which contains the Polar to Rectangular routines XEQ Z015 on line R049 is the Polar→Rectangular conversion

USER INSTRUCTIONS COORDINATE JOINS PROGRAM

1. To start program press **XEQ** J001

2.	Display	E? 0.0000	Enter:	East coordinate of Instrument Point; then press R/S
3.	Display	N? 0.0000	Enter:	North coordinate of Instrument Point; then press R/S
4.	Display	E? 0.0000	Enter:	East coordinate of next point; then press R/S
5.	Display	N? 0.0000	Enter: [If next	North coordinate of next point; then press R/S Instrument Point, enter Northing with negative sign.]

6. Bearing (D.MMSS) and Distance displayed at successive R/S. GoTo step 4.

In the example traverse below, start at A, compute the radiations (bearings and distances) to A1 and A2; jump to B, compute radiations to B1 and B2; then to C and the radiations to C1, C2 and C3. (The computed bearings and distances are rounded to the nearest 5 mm and 10" respectively.)



HP35s PROGRAM SHEET JOINS PROGRAM

LINE	STEP	STEP X Y		Z	Т
J001	LBL J	LBL J			
J002	CLVARS		START JOIN	S PROGRAM	
J003	INPUT E	Enter East	c coord of	Instrument	Point.
J004	INPUT N	Enter Nort	ch coord of	I.P.	
J005	RCL E	Ei	NEW INSTRU	MENT POINT	
J006	STO Y				
J007	RCL N	Ni	Ei		
J008	STO X				
J009	CF 1		NEW POINT		
J010	0				
J011	STO E				
J012	STO N				
J013	INPUT E	Enter East	c coord of	next point	
J014	INPUT N	Enter ±Nor	th coord o	f next poir	nt
J015	<i>x</i> < 0 ?	$\pm N_k$	E _k		
J016	SF 1				
J017	ABS	N _k			
J018	STO N				
J019	RCL Y	Ei			
J020	RCL E	E _k	Ei		
J021	-	Ei-Ek			
J022	RCL X	Ni	E _i -E _k		
J023	RCL N	N _k	Ni	E _i -E _k	
J024	-	N _i - N _k	E _i -E _k		
J025	XEQ Z002				
J026	STO D	Dist	Brg(k,i)		
J027	x <> y	Brg(k,i)			
J028	180				
J029	+	Brg(i,k)			
J030	→HMS				
J031	STO B				
J032	VIEW B	Bearing			
J033	VIEW D	Distance			
J034	FS? 1				
J035	GTO J005				
J036	GTO J009				

STORAGE REGISTERS

В	Bearing(D.MMSS)
D	Distance
E	E _k East coordinate
Ν	N _k North coordinate
X	N _i North coordinate of Instrument Point
Y	E _i East coordinate of Instrument Point

PROGRAM LENGTH AND CHECKSUM

LN = 112; CK = A366

*	Length	&	Checksum:	\leftarrow	\checkmark
---	--------	---	-----------	--------------	--------------

	-				
] 2	2	;	$\left \leftarrow \right $	ENTER	(Hold)

PROGRAM NOTES

Flag 1 is used to test to see if new point is to be next Instrument Point. Lines J001 to J004 Initialisation; storing coordinates of Instrument Point. Lines J005 to J008 Storing coordinates of Instrument Point in registers X and Y. Lines J009 to J014 Entering coordinates of next point. Lines J015 to J036 Computing Bearing and Distance from Instrument Point to new point. If Flag 1 is set then the new point becomes the next Instrument Point.

The calculator must contain LBL Z which contains the Polar to Rectangular routines

XEQ Z002 on line J025 is the Rectangular \rightarrow Polar conversion

SHEET 2 OF 2 SHEETS

USER INSTRUCTIONS OFFSETS PROGRAM

1. To start program press **XEQ O001**

2.	Display	B? 1.0000	Enter:	B_1 , the bearing of traverse line 1; then press R/S
3.	Display	B? 2.0000	Enter:	B_2 , the bearing of traverse line 2; then press R/S
4.	Display	D? 11.0000	Enter:	$\pm d_1$, the offset from traverse line 1; then press R/S
5.	Display	D? 22.0000	Enter:	$\pm d_2$, the offset from traverse line 2; then press R/S

6. Radiation Bearing (D.MMSS) B_3 and Distance d_3 displayed at successive R/S. GoTo step 2.

Rule: Offset distances are $\begin{cases} positive \\ negative \end{cases}$ if point is to the $\begin{cases} right \\ left \end{cases}$ of the traverse line looking in the direction of the bearing.

Derivation of formula: Radiation from offsets



Conventions: $\theta = B_2 - B_1$ $d \text{ is } \begin{cases} +^{\text{tve}} \\ -^{\text{tve}} \end{cases}$ if point is $\begin{cases} right \\ left \end{cases}$ of line $B_3 = B_1 + \alpha$

Formula:

$$\tan \alpha = \frac{\sin \theta}{\cos \theta - \frac{d_2}{d_1}}$$

HP35s PROGRAM SHEET OFFSETS PROGRAM

LINE	STEP	X	Y	Z	Т
0001	LBL O				
0002	1				
0003	STO B	1			
0004	INPUT B	Enter Bear	ring of 1st	line (B_1)	1
0005	HMS→	B ₁			
0006	STO A				
0007	2				
0008	STO B	2			
0009	INPUT B	Enter Bear	ring of 2nd	line (B_2)	
0010	HMS→	B ₂			
0011	RCL A	B ₁	B ₂		
0012	-	$\pm \theta = B_2 - B_1$			
0013	360	360	±θ		
0014	<i>x</i> <> <i>y</i>	$\pm \theta$	360		
0015	<i>x</i> < 0 ?				
0016	+				
0017	STO T	θ			
0018	11				
0019	STO D	11			
0020	INPUT D	Enter Offs	set $\pm d_1$ from	n 1st line	
0021	STO C	$\pm d_1$			
0022	22				
0023	STO D	22			
0024	INPUT D	Enter Offs	set $\pm d_2$ from	m 2nd line	
0025	RCL T	θ			
0026	SIN	$sin(\theta)$	$\pm d_2$		
0027	RCL T	θ	$sin(\theta)$	$\pm d_2$	
0028	COS	$\cos(\theta)$	$sin(\theta)$	$\pm d_2$	
0029	RCL D	+d ₂	$\cos(\theta)$	$sin(\theta)$	$+d_2$
0030	RCL C	+d1	+d ₂	$COS(\theta)$	$sin(\theta)$
0031	•	$+d_{0}/d_{1}$	$-\alpha_2$	sin(A)	$sin(\theta)$
0032	-	$\pm \alpha_2 / \alpha_1$	cos(0)	sin(0)	$\sin(\theta)$
0032		$t = t = (\alpha_2) \alpha_1$	5111(0)	5111(0)	SIII(0)
0033	÷	$tan(\alpha)$			
0034	AIAN	$\pm \alpha$			
0035	SIU+A	В3			
0030	SIN STO S	$sin(\alpha)$			
	STO÷C				
0038	RCL C	±d ₃			
0039	0	0	±d ₃		
0040	x > y?				
0041	180				
0042	STO+A				

SHEET 1 OF 2 SHEETS

HP35s PROGRAM SHEET OFFSETS PROGRAM

LINE	STEP	X	Y	Z	Т
0043	RCL A	B ₃			
0044	360	360	B ₃		
0045	x < y?				
0046	STO-A				
0047	RCL A	B ₃			
0048	→HMS				
0049	STO B				
0050	VIEW B	Radiation	Bearing B_3		
0051	RCL C	$\pm d_3$			
0052	ABS				
0053	STO D				
0054	VIEW D	Radiation	<i>Distance</i> d	3	
0055	GTO 0002				

STORAGE REGISTERS

Α	B ₁ ; B ₃
В	$B_2(D.MMSS); B_3(D.MMSS)$
С	$\pm d_1; \pm d_3$
D	$\pm d_2; d_3$
Т	θ

PROGRAM LENGTH AND CHECKSUM

LN = 181; CK = A802

★ Length & Checksum: $\square \square \square 2$; $\square \square \square$ (Hold)



SHEET 2 OF 2 SHEETS

USER INSTRUCTIONS RESECTION PROGRAM (Auxiliary angles method)

1.	To start p	rogram press	XEQ S	001
2.	Display	E? 1.0000	Enter:	East coordinate of Point 1 (P ₁); then press \blacksquare
3.	Display	N? 1.0000	Enter:	North coordinate of P_1 ; then press R/S
4.	Display	E? 2.0000	Enter:	East coordinate of P_2 ; then press R/S
5.	Display	N? 2.0000	Enter:	North coordinate of P_2 ; then press R/S
6.	Display	E? 3.0000	Enter:	East coordinate of P_3 ; then press R/S
7.	Display	N? 3.0000	Enter:	North coordinate of P_3 ; then press R/S
8.	Display	A? 0.0000	Enter:	Angle α (D.MMSS) at Resection Point P; then press R/S
9.	Display	B? 0.0000	Enter:	Angle β (D.MMSS) at Resection Point P; then press R/S

10. East and North coordinate of Resection Point displayed at successive R/S. GoTo step 2.

- Notes: (1) Coordinates of points P_1 , P_2 , P_3 must be entered left to right (clockwise direction) as seen from the Resection Point P.
 - (2) Observed angles α and β are angles P₁-P-P₂ and P₂-P-P₃ respectively.

EXAMPLE



RESECTION – AUXILIARY ANGLES



In each case there is a four-sided figure *P123* with angles $(\alpha + \beta)$, φ , γ and ψ at the vertices. φ and ψ are unknown 'auxiliary angles' and $\varphi + \psi = 360^{\circ} - (\alpha + \beta + \gamma)$. α and β are known (observed) and γ is the difference in bearings of lines 2–1 and 2–3.

GIVEN:	1 (E ₁ , N ₁), 2 (E ₂ , N ₂), 3 (E ₃ , N ₃)
OBSERVED:	α, β
COMPUTE:	\boldsymbol{P} (E _P , N _P)

3.

- 1. Compute bearings and distances of lines 2-1 and 2-3
- 2. Calculate angle γ as the difference between bearings B_{21} and B_{23} . [B_{KJ} means the bearing from K to J]

$$\varphi + \psi = 360^{\circ} - (\alpha + \beta + \gamma) = \theta \tag{1}$$

4. From sine rule: $\frac{d_{2P}}{\sin \varphi} = \frac{d_{21}}{\sin \alpha} \quad \text{or} \quad d_{2P} = \frac{d_{21} \sin \varphi}{\sin \alpha}$

$$\frac{d_{2P}}{\sin\psi} = \frac{d_{23}}{\sin\beta} \quad \text{or} \quad d_{2P} = \frac{d_{23}\sin\psi}{\sin\beta}$$
(3)

CASE 3

Р

(2)

HP35s PROGRAM

Equating (2) and (3) gives

$$\frac{\sin\varphi}{\sin\psi} = \frac{d_{23}\sin\alpha}{d_{21}\sin\beta} = a \tag{4}$$

5. From (4) $\sin \varphi = a \sin \psi$, but from (1) $\psi = \theta - \varphi$; hence $\sin \varphi = a \sin(\theta - \varphi) = a(\sin \theta \cos \varphi - \cos \theta \sin \varphi)$. Dividing both sides by $\cos \varphi$ and re-arranging gives $\tan \varphi (1 + a \cos \theta) = a \sin \theta$ and

$$\tan \varphi = \frac{a \sin \theta}{1 + a \cos \theta} \tag{5}$$

- 6. After computing θ [using (1)], a [using (4)] and φ [using (5)] then ψ can be calculated using (1).
- 7. The bearing B_{1P} (bearing of the line 1-P) is given by

$$B_{1P} = B_{12} + \varphi \tag{6}$$

The distance d1P (distance of line 1-P) is obtained using the *sine rule* in triangle 12P and

$$d_{1P} = \frac{d_{12}\sin(\alpha + \varphi)}{\sin\alpha} \tag{7}$$

8. E_P and N_P obtained from E_1 and N_1 and the bearing B_{1P} and distance d_{1P} of the line 1-P.

EXAMPLE



HP35s PROGRAM SHEET RESECTION PROGRAM

LINE	STEP	X	Y	Z	T
S001	LBL S				
S002	CLVARS		START RESE	CTION PROG	RAM
S003	1				
S004	XEQ S093	Enter coor	dinates of	Point 1	
S005	RCL E	E ₁			
S006	STO R	E ₁			
S007	RCL N	N ₁			
S008	STO U	N ₁	E ₁		
S009	2				
S010	XEQ S093	Enter cooi	rdinates of	Point 2	
S011	RCL E	E ₂			
S012	STO S	E ₂			
S013	RCL N	N ₂			
S014	STO V	N ₂	보 ₂		
S015	3				
SU16 S017	XEQ SU93	Enter cooi	rainates of	Point 3	
SU17	INPUI A	Enter ang.	le α (D.MMS	5)	
SU10 C010	HMS-	α			
5019	JIU A				
5020	INPULB	Enter ang.	le p (D.MMS	S)	
SUZI	HMS→	-			
S022	STO B	β			
S023	RCL S	E ₂			
S024	RCL E	E 3	E ₂		
S025	-	$\Delta E_{32} = E_2 - E_3$			
S026	RCL V	N ₂	Δ E ₃₂		
S027	RCL N	N 3	N ₂	Δ E ₃₂	
S028	-	$\Delta N_{32} = N_2 - N_3$	Δ E ₃₂		
S029	XEQ Z002	$d_{32} = d_{23}$	B ₃₂		
S030	RCL A	α	d ₂₃	B ₃₂	
S031	SIN	$sin(\alpha)$	d ₂₃	B ₃₂	
S032	×	$d_{23}sin(\alpha)$	B ₃₂		
S033	STO C				
S034	x <> y	B ₃₂	$d_{23}sin(\alpha)$		
S035	180	180	B ₃₂	$d_{23}sin(\alpha)$	
S036	+	B ₂₃	$d_{23}sin(\alpha)$		
S037	RCL S	E ₂	B ₃₂	$d_{23}sin(\alpha)$	
S038	RCL R	E ₁	E ₂	B ₃₂	$d_{23}sin(\alpha)$
S039	-	$\Delta E_{12} = E_2 - E_1$	B ₃₂	$d_{23}sin(\alpha)$	$d_{23}sin(\alpha)$
S040	RCL V	N ₂			
S041	RCL U	N ₁	N ₂	ΔE_{12}	B ₃₂
S042	-	$\Delta \overline{N_{12}} = \overline{N_2} - \overline{N_1}$	ΔE_{12}	B ₃₂	B ₃₂

SHEET 1 OF 4 SHEETS

HP35s PROGRAM SHEET RESECTION PROGRAM

LINE	STEP	X	Y	Z	Т
S043	XEQ Z002	d ₁₂	B ₁₂	B ₃₂	B ₃₂
S044	STO D				
S045	RCL B	β	d ₁₂	B ₁₂	B ₃₂
S046	SIN	$sin(\beta)$	d ₁₂	B ₁₂	B ₃₂
S047	×	$d_{12}sin(\beta)$	B ₁₂	B ₃₂	B ₃₂
S048	STO÷C				
S049	R↓	B ₁₂	B ₃₂	B ₃₂	$d_{12}sin(\beta)$
S050	STO F				
S051	180	180	B ₁₂	B ₃₂	B ₃₂
S052	+	B ₂₁	B ₃₂	B ₃₂	B ₃₂
S053	x <> y	B ₃₂	B ₂₁	B ₃₂	B ₃₂
S054	-	±γ	B ₃₂	B ₃₂	B ₃₂
S055	360				
S056	x <> y	±γ	360	B ₃₂	B ₃₂
S057	<i>x</i> < 0 ?				
S058	+	γ			
S059	RCL A	α	γ		
S060	RCL B	β	α	γ	
S061	+			•	
S062	+	$\alpha + \beta + \gamma$			
S063	360				
S064	<i>x</i> <> <i>y</i>				
S065	-	$\theta = 360 - (\alpha + \beta)$	$(\beta + \gamma)$		
S066	RCL C	a	θ		
S067	XEQ Z015	$a \times \cos(\theta)$	$a \times sin(\theta)$		
S068	1				
S069	+	$1 + acos(\theta)$	$asin(\theta)$		
S070	XEQ Z002		φ		
S071	<i>x</i> <> <i>y</i>	φ			
S072	STO+F	φ			
S073	RCL A	α	φ		
S074	+	$\alpha + \phi$			
S075	SIN	$sin(\alpha + \phi)$			
S076	STO×D				
S077	RCL A	α			
S078	SIN	$sin(\alpha)$			
S079	STO÷D				
S080	RCL F	B _{1P}		1	
S081	RCL D	d _{1P}	B _{1P}		
S082	XEQ Z015	ΔN_{1P}	ΔE_{1P}		

SHEET 2 OF 4 SHEETS

HP35s PROGRAM SHEET

RESECTION PROGRAM

LINE	STEP	Х	Y	Z	Т
S083	RCL U	N ₁	Δ N _{1P}	ΔE_{1P}	
S084	+	$N_{P} = N_{1} + \Delta N_{1P}$	ΔE_{1P}		
S085	STO N				
S086	x <> y	ΔE_{1P}	N_{P}		
S087	RCL R	E ₁	ΔE_{1P}	N_{P}	
S088	+	$\mathbf{E}_{P} = \mathbf{E}_{1} + \Delta \mathbf{E}_{1P}$	N _P		
S089	STO E				
S090	VIEW E	$E_P = East$	coordinate	of resecte	d point
S091	VIEW N	N_P = North	coordinate	e of resect	ed point
S092	GTO S002				
S093	STO E	1,2,3			
S094	STO N				
S095	INPUT E	Enter ${\tt E}_{\tt k}$ (k = 1, 2, 3)		
S096	INPUT N	Enter N_k (k = 1, 2, 3)		
S097	RTN				

STORAGE REGISTERS

Α	α
в	β
С	$d_{23}\sin(\alpha)$; $a=d_{23}\sin(\alpha)/d_{12}\sin(\beta)$
D	d_{12} ; $d_{12}\sin(\alpha+\phi)$; $d_{1P}=[d_{12}\sin(\alpha+\phi)]/\sin(\alpha)$
F	B_{12} ; $B_{1P} = (B_{12} + \varphi)$
E	\mathbf{E}_{k} ; \mathbf{E}_{3} ; \mathbf{E}_{P}
Ν	N_k ; N_3 ; N_P
R	E ₁
U	N ₁
S	E ₂
V	N ₂

PROGRAM LENGTH AND CHECKSUM

LN = 307; CK = AE78

* Length & Checksum: $\square \square \square 2$; $\square \square \square$ (Hold)

SHEET 3 OF 4 SHEETS

HP35s PROGRAM SHEET

PROGRAM NOTES

P₁, P₂, P₃ means Points 1, 2 and 3. E₁, E₂, etc. and N₁, N₂, etc. mean east and north coordinates of P₁, P₂, etc. $\Delta E_{12}=E_2-E_1$, $\Delta N_{12}=N_2-N_1$, etc. B₁₂ means bearing of the line from P₁ to P₂ d₁₂ means distance from P₁ to P₂

```
Lines S001 to S016
                          Initialisation; storing coordinates of
                          P_1, P_2, P_3.
Lines S017 to S022
                          Entering and storing observed angles
                          \alpha and \beta at the Resection Point P.
Lines S023 to S029
                         Bearing and distance P_3 to P_2.
Lines S034 to S036
                         Note here that B_{23} = B_{32} + 180^{\circ}
Lines S037 to S043
                         Bearing and distance P_1 to P_2.
Lines S044 to S065
                          Calculation of angles \gamma at P<sub>2</sub> and
                          \theta=360°-(\alpha+\beta+\gamma); and the ratio
                          a=d_{23}sin(\alpha)/d_{12}sin(\beta)
                          Calculation of auxiliary angle \boldsymbol{\phi} and
Lines S066 to S079
                          the bearing and distance P_1 to the
                          resection Point P: B_{1P}=(B_{12}+\phi) and
                          d_{1P} = [d_{12}\sin(\alpha + \phi)] / \sin(\alpha).
Lines S080 to S091
                         Calculation and display of coordinates
                          of Resected Point P.
Lines S093 to S097
                         Subroutine for entering coordinates of
                          P_1, P_2, P_3.
```

The calculator must contain LBL Z which contains the Polar to Rectangular routines XEQ Z002 on lines S067, S082 is the Rectangular→Polar conversion XEQ Z015 on lines S029, S043, S070 is the Polar→Rectangular conversion

SHEET 4 OF 4 SHEETS

USER INSTRUCTIONS TRAVERSE ADJUSTMENT PROGRAM

This program can perform either a BOWDITCH¹ or a CRANDALL² adjustment on a <u>closed</u> <u>traverse</u> (or figure). The bearings and distances of each line of the closed traverse must be entered before selecting the method of adjustment (1 = Bowditch; 2 = Crandall).

After all lines have been entered and adjustment type selected the program will display the adjusted bearings and distances and then the area of the adjusted figure.

A <u>closed traverse</u> must start and end at known points (east and north coordinates known); but in the case of a loop traverse the start and end points will be the same. The program requires that $D_E = E_{END} - E_{START}$ and $D_N = N_{END} - N_{START}$ are known. If the traverse is a loop traverse $D_E = D_N = 0$

1. To start program press **XEQ** A001

2.	Display	B ?	Enter: Bearing (D.MMSS); then press R/S
		0.0000	[Bearing of lines that are 0° 00' 00" must be entered as 360° 00' 00'
3.	Display	D? 0.0000	Enter: Distance; then press R/S

4. Repeat steps 2 and 3 until all known information is entered; then enter 0 at the Bearing prompt and 0 at the Distance prompt (just press R/S at the prompts)

5.	Display	X? 0.0000	Enter: D_E ; then press R/S [If loop traverse $D_E = 0$, just press R/S]
6.	Display	Y? 0.0000	Enter: D_N ; then press R/S [If loop traverse $D_N = 0$, just press R/S]
7.	Display	F? 0.0000	Enter: $1 = BOWDITCH \text{ or } 2 = CRANDALL; \text{ then press}$

- 8. Adjusted Bearings (D.MMSS) and adjusted Distances displayed at successive R/S. [Note that Crandall's adjustment only adjusts distances]
- 9. Adjusted Area displayed at last prompt. Press R/S and go to step 2 for new adjustment.

¹ A mathematical adjustment of chain and compass surveys developed by the American mathematician and astronomer Nathaniel Bowditch (1773-1838). This adjustment affects both bearings and distances.

² A mathematical 'least squares' adjustment of traverse distances only that assumes that observed bearings 'close' perfectly. Developed in 1906 by Charles L. Crandall, Professor of Railroad Engineering and Geodesy, Cornell University, New York.

Theory and examples of Bowditch's and Crandall's adjustments can be found in *Notes on Least Squares*, Geospatial Science, RMIT University, Chapter 6, pp.6-15 – 6-26.

THEORY AND FORMULA

Theory, formula and examples of Bowditch's and Crandall's adjustments can be found in *Notes on Least Squares*, Geospatial Science, RMIT University, Chapter 6, pp.6-15 – 6-26. A summary of the formula and the sequence of computation is presented below.

BOWDITCH

A closed traverse of k = 1, 2, 3..., n lines, sides or legs having bearings ϕ_k and distances d_k (or a figure of *n* sides) that has a misclosure may be adjusted in the following manner.

1. Each traverse line (having bearing and distance) has east and north components $\Delta E_k = d_k \sin \phi_k$, $\Delta N_k = d_k \cos \phi_k$, and the sums of these components for the traverse are

$$S_E = \sum_{k=1}^n \Delta E_k$$
 and $S_N = \sum_{k=1}^n \Delta N_k$

- 2. A traverse has a total length $L = \sum_{k=1}^{n} d_k$
- 3. A closed traverse has a start point and an end point assumed to have known east and north coordinates; E_{START} , N_{START} , E_{END} , N_{END} and differences; $D_E = E_{END} E_{START}$ and $D_N = N_{END} N_{START}$. If the traverse is a loop traverse (starting and ending at the same point), then $D_E = D_N = 0$.
- 4. The east and north components of each traverse leg may be adjusted by adding corrections

$$dE_{k} = d_{k} \left(\frac{D_{E} - S_{E}}{L}\right) \text{ and } dN_{k} = d_{k} \left(\frac{D_{N} - S_{N}}{L}\right) \text{ so that } \left\{\frac{\Delta E_{k}}{\Delta N_{k}}\right\}_{ADJUST} = \left\{\frac{\Delta E_{k}}{\Delta N_{k}}\right\}_{OBS} + \left\{\frac{dE_{k}}{dN_{k}}\right\}_{OBS}$$

5. Adjusted bearings and distances and area are then computed from the adjusted east and north components.

CRANDALL

A closed traverse of k = 1, 2, 3..., n lines, sides or legs having bearings ϕ_k and distances d_k (or a figure of *n* sides) that has a misclosure may be adjusted in the following manner.

- 1. First adjust the bearings of the traverse so that they close perfectly. This may be an arbitrary adjustment.
- 2. Each traverse line (having bearing and distance) has east and north components $\Delta E_k = d_k \sin \phi_k$, $\Delta N_k = d_k \cos \phi_k$, and the sums of these components for the traverse are

$$S_E = \sum_{k=1}^n \Delta E_k$$
 and $S_N = \sum_{k=1}^n \Delta N_k$

THEORY AND FORMULA continued

3. In addition, the traverse has the following summations: $a = \sum_{k=1}^{n} \frac{(\Delta E_k)^2}{d_k}, b = \sum_{k=1}^{n} \frac{(\Delta N_k)^2}{d_k}$, and

$$c = \sum_{k=1}^{n} \frac{\Delta E_k \Delta N_k}{d_k}$$

4. A closed traverse has a start point and an end point assumed to have known east and north coordinates; E_{START} , N_{START} , E_{END} , N_{END} and differences; $D_E = E_{END} - E_{START}$ and $D_N = N_{END} - N_{START}$. If the traverse is a loop traverse (starting and ending at the same point), then $D_E = D_N = 0$.

5. Two 'multipliers' are computed:
$$\begin{cases} k_1 \\ k_2 \end{cases} = \begin{cases} \frac{b(D_E - S_E) - c(D_N - S_N)}{ab - c^2} \\ \frac{a(D_N - S_N) - c(D_E - S_E)}{ab - c^2} \end{cases}$$

6. A residual v_k for each traverse line is computed from $v_k = k_1 \Delta E_k + k_2 \Delta N_k$ and added to the observed traverse distance to obtain the adjusted traverse distance: $d_{ADJUST} = d_{OBS} + v$

EXAMPLE 1



Figure 1. Fieldnotes of traverse

Figure 1 shows a traverse between points *A*, *B*, *C* and *D*. The bearing datum of the survey is the line *AB* 285° 00' 00". The distances are horizontal distances. Observed face-left (FL) bearings are shown along the traverse line and the seconds part of the face-right (FR) bearing is shown above. The mean of the FL/FR seconds is shown to the right of the brace $\}$. The angular misclose in the traverse is 20", which is revealed in the forward and reverse bearings on the line *CD*.

1. BOWDITCH ADJUSTMENT OF EXAMPLE 1

For the purpose of the exercise we assume that the angular misclose of 20" is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

Line	Bearing	Distance	components		corrections		adjusted components	
k	$\pmb{\phi}_k$	d_k	ΔE_k	ΔN_k	dE_k	dN_k	ΔE_k	ΔN_k
1: AB	285° 00' 00"	268.786	-259.6273	69.5669	0.0068	-0.0139	-259.6205	69.5530
2: BC	346° 37′ 29″	156.627	-36.2322	152.3786	0.0040	-0.0081	-36.2282	152.3705
3: CD	93° 42′ 25″	148.650	148.3390	-9.6107	0.0038	-0.0077	148.3428	-9.6184
4: DA	145° 12′ 31″	258.503	147.4993	-212.2917	0.0066	-0.0134	147.5059	-212.3051
	sums	832.5660	-0.0212	0.0431	0.0212	-0.0431	0.0000	0.0000

$$L = \sum_{k=1}^{n} d_k = 832.5660, \ S_E = \sum_{k=1}^{n} \Delta E_k = -0.0212 \text{ and } S_N = \sum_{k=1}^{n} \Delta N_k = 0.0431$$

Since this is a loop traverse $D_E = D_N = 0$ and $D_E - S_E = 0.0212$, $D_N - S_N = -0.0431$

The corrections to the traverse components are:

$$dE_{k} = d_{k} \left(\frac{D_{E} - S_{E}}{L}\right) = d_{k} \left(\frac{0.0212}{832.5660}\right)$$
$$dE_{k} = d_{k} \left(\frac{D_{N} - S_{N}}{L}\right) = d_{k} \left(\frac{-0.0431}{832.5660}\right)$$

The adjusted traverse is

Line	Bearing	Distance
k	$\pmb{\phi}_k$	d_k
1: AB	284° 59′ 51″	268.7758
2: BC	346° 37′ 32″	156.6182
3: CD	93° 42′ 35″	148.6543
4: DA	145° 12′ 33″	258.5178

Using the program: press XEQ A001 (or XEQ A ENTER)

Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry

When all sides have been keyed in, enter 0 at the prompt B? and press R/S; and 0 at the prompt D? and press R/S (or simply press R/S at both prompts).

At the prompt X? enter 0 and press R/S ($D_E = 0$) and at the prompt Y? enter 0 and press R/S ($D_N = 0$)

At the prompt F? enter 1 and press R/S.

The calculator will then display the adjusted bearing at B =. Press R/S and the adjusted distance will be displayed at D =. Repeat pressing of R/S will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $A = (The area = -33,556.9387 m^2)$

2. CRANDALL ADJUSTMENT OF EXAMPLE 1

For the purpose of the exercise we assume that the angular misclose of 20" is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

Line	Bearing	Distance	components		$(\Delta E_{\rm r})^2$	$(\Delta N_{\cdot})^2$	$\Delta E_{\mu} \Delta N_{\mu}$	residual
k	$\pmb{\phi}_k$	d_k	ΔE_k	ΔN_k	$\frac{(\underline{-}_{k})}{d_{k}}$	$\frac{(\underline{-},\underline{k})}{d_k}$	$\frac{k}{d_k}$	v_k
1: AB	285° 00' 00"	268.786	-259.6273	69.5669	250.7808	18.0052	-67.1965	-0.004
2: BC	346° 37′ 29″	156.627	-36.2322	152.3786	8.3815	148.2455	-35.2495	-0.021
3: CD	93° 42′ 25″	148.650	148.3390	-9.6107	148.0286	0.6214	-9.5906	-0.002
4: DA	145° 12′ 31″	258.503	147.4993	-212.2917	84.1616	174.3414	-121.1316	0.027
	sums	832.5660	-0.0212	0.0431	491.3526	341.2134	-233.1681	

$$S_{E} = \sum_{k=1}^{n} \Delta E_{k} = -0.0212, \ S_{N} = \sum_{k=1}^{n} \Delta N_{k} = 0.0431$$
$$a = \sum_{k=1}^{n} \frac{\left(\Delta E_{k}\right)^{2}}{d_{k}} = 491.3526, \ b = \sum_{k=1}^{n} \frac{\left(\Delta N_{k}\right)^{2}}{d_{k}} = 341.2134 \text{ and } c = \sum_{k=1}^{n} \frac{\Delta E_{k} \Delta N_{k}}{d_{k}} = -233.1681$$

Since this is a loop traverse $D_E = D_N = 0$ and $D_E - S_E = 0.0212$, $D_N - S_N = -0.0431$

The multipliers are:
$$k_1 = \frac{b(D_E - S_E) - c(D_N - S_N)}{ab - c^2} = -2.4593e - 05$$

 $k_2 = \frac{a(D_N - S_N) - c(D_E - S_E)}{ab - c^2} = -1.4324e - 04$

The residuals are: $v_k = k_1 \Delta E_k + k_2 \Delta N_k$

The adjusted traverse (nearest mm) is

Line	Bearing	Distance
k	$\pmb{\phi}_k$	d_k
1: AB	285° 00' 00"	268.782
2: BC	346° 37′ 29″	156.606
3: CD	93° 42′ 25″	148.648
4: DA	145° 12′ 31″	258.530

Using the program: press **XEQ** A001 (or **XEQ** A **ENTER**)

Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry

When all sides have been keyed in, enter 0 at the prompt B? and press R/S; and 0 at the prompt D? and press R/S (or simply press R/S at both prompts).

At the prompt X? enter 0 and press R/S ($D_E = 0$) and at the prompt Y? enter 0 and press R/S ($D_N = 0$)

At the prompt F? enter 2 and press R/S.

The calculator will then display the adjusted bearing at B =. Press R/S and the adjusted distance will be displayed at D =. Repeat pressing of R/S will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $A = (The area = -33,555.9331 \text{ m}^2)$

EXAMPLE 2



Figure 2 Traverse diagram showing field measurements, derived values and fixed values.

Figure 2 is a schematic diagram of a traverse run between two fixed stations *A* and *B* and oriented at both ends by angular observations to a third fixed station *C*.

The bearings of traverse lines shown on the diagram, unless otherwise indicated, are called "observed" bearings and have been derived from the measured angles (which have been derived from observed theodolite directions) and the fixed bearing AC. The difference between the observed and fixed bearings of the line BC represents the angular misclose of 15". The coordinates of the traverse points D, E and F have been calculated using the observed bearings and distances and the fixed coordinates of A. The difference between the observed and fixed coordinates at B represents a traverse misclosure.

3. BOWDITCH ADJUSTMENT OF EXAMPLE 2

For the purpose of the exercise we assume that the angular misclose of 15" is acceptable and that this error is apportioned equally at the five traverse points giving the observed traverse to be adjusted as shown in the left-columns of the table below

Line	Bearing	Distance	components		corrections		adjusted components	
k	$\pmb{\phi}_k$	d_k	ΔE_k	ΔN_k	dE_k	dN_k	ΔE_k	ΔN_k
1: AD	110° 15′ 17″	2401.609	2253.1002	-831.4235	-0.0029	-0.0994	2253.0973	-831.5229
2: DE	68° 34′ 12″	1032.340	960.9688	377.1801	-0.0012	-0.0427	960.9676	377.1374
3: EF	163°03′ 23″	559.022	162.9160	-534.7560	-0.0007	-0.0231	162.9153	-534.7791
4: FB	113° 49′ 38″	1564.683	1431.3217	-632.1006	-0.0019	-0.0648	1431.3198	-632.1654
	sums	5557.6540	4808.3067	-1621.1000	-0.0067	-0.2300	0.0000	0.0000

$$L = \sum_{k=1}^{n} d_k = 5557.65400$$
, $S_E = \sum_{k=1}^{n} \Delta E_k = 4808.3067$ and $S_N = \sum_{k=1}^{n} \Delta N_k = -1621.1000$

 $D_{\rm E} = E_{\rm END} - E_{\rm START} = 6843.085 - 2034.785 = 4808.300$

 $D_{N} = N_{END} - N_{START} = 7154.700 - 8776.030 = -1621.330$

 $D_E - S_E = -0.0067$, $D_N - S_N = -0.2300$

The corrections to the traverse components are:

$$dE_{k} = d_{k} \left(\frac{D_{E} - S_{E}}{L}\right) = d_{k} \left(\frac{-0.0067}{5557.6540}\right)$$
$$dE_{k} = d_{k} \left(\frac{D_{N} - S_{N}}{L}\right) = d_{k} \left(\frac{-0.2300}{5557.6540}\right)$$

The adjusted traverse is

Line	Bearing	Distance
k	$\pmb{\phi}_k$	d_k
1: AD	110° 15′ 25″	2401.6407
2: DE	68° 34′ 20″	1032.3232
3: EF	163° 03′ 26″	559.0439
4: FB	113° 49′ 46″	1564.7074

Using the program: press **XEQ** A001 (or **XEQ** A **ENTER**)

Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry

When all sides have been keyed in, enter 0 at the prompt B? and press R/S; and 0 at the prompt D? and press R/S (or simply press R/S at both prompts).

At the prompt X? enter 4808.300 and press R/S ($D_E = 4808.300$)

At the prompt Y? enter -1621.330 and press R/S ($D_N = -1621.330$)

At the prompt F? enter 1 and press R/S.

The calculator will then display the adjusted bearing at B =. Press R/S and the adjusted distance will be displayed at D =. Repeat pressing of R/S will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $A = (The area = -357,496.7606 m^2 but is meaningless since this is not a closed polygon)$

4. CRANDALL ADJUSTMENT OF EXAMPLE 2

For the purpose of the exercise we assume that the angular misclose of 20" is acceptable and that this error is apportioned equally at the four corners giving the observed traverse to be adjusted as shown in the left-columns of the table below

Line	Bearing	Distance	components		$(\Delta E_{\rm e})^2$	$(\Delta N_{\star})^2$	$\Delta E_{i} \Delta N_{i}$	resid.
k	$\pmb{\phi}_k$	d_k^{-}	ΔE_k	ΔN_k	$\frac{(_{k})}{d_{k}}$	$\frac{(,,,)}{d_k}$	$\frac{d_k}{d_k}$	v_k
1: AD	110° 15′ 17″	2401.609	2253.1002	-831.4235	2113.7748	287.8342	-780.0106	0.057
2: DE	68° 34′ 12″	1032.340	960.9688	377.1801	894.5319	137.8081	351.1036	-0.168
3: EF	163°03′ 23″	559.022	162.9160	-534.7560	47.4788	511.5433	-155.8442	0.129
4: FB	113° 49′ 38″	1564.683	1431.3217	-632.1006	1309.3270	255.3560	-578.2253	0.064
	sums	5557.6540	4808.3067	-1621.1000	4365.1124	1192.5416	-1162.9764	

$$\begin{split} S_E &= \sum_{k=1}^n \Delta E_k = 4808.3067 , \ S_N = \sum_{k=1}^n \Delta N_k = -1621.1000 \\ a &= \sum_{k=1}^n \frac{\left(\Delta E_k\right)^2}{d_k} = 4365.1124 , \ b = \sum_{k=1}^n \frac{\left(\Delta N_k\right)^2}{d_k} = 1192.5416 \ \text{and} \ c = \sum_{k=1}^n \frac{\Delta E_k \Delta N_k}{d_k} = -1162.9764 \\ D_E &= E_{END} - E_{START} = 6843.085 - 2034.785 = 4808.300 \\ D_N &= N_{END} - N_{START} = 7154.700 - 8776.030 = -1621.330 \\ D_E &- S_E = -0.0067 , \ D_N - S_N = -0.2300 \end{split}$$

The multipliers are:
$$k_1 = \frac{b(D_E - S_E) - c(D_N - S_N)}{ab - c^2} = -7.1501e - 05$$

 $k_2 = \frac{a(D_N - S_N) - c(D_E - S_E)}{ab - c^2} = -2.6259e - 04$

The residuals are: $v_k = k_1 \Delta E_k + k_2 \Delta N_k$

The adjusted traverse (nearest mm) is

Line	Bearing	Distance
k	$\pmb{\phi}_k$	d_k
1: AB	110° 15′ 17″	2401.666
2: BC	68° 34′ 12″	1032.172
3: CD	163°03′ 23″	559.151
4: DA	113° 49′ 38″	1564.747

HP35s PROGRAM

Using the program: press **XEQ** A001 (or **XEQ** A **ENTER**)

Enter the bearings and distances of the sides at the prompts B? and D? pressing R/S after entry

When all sides have been keyed in, enter 0 at the prompt B? and press R/S; and 0 at the prompt D? and press R/S (or simply press R/S at both prompts).

At the prompt X? enter 4808.300 and press R/S ($D_E = 4808.300$)

At the prompt Y? enter -1621.330 and press R/S ($D_N = -1621.330$)

At the prompt F? enter 2 and press R/S.

The calculator will then display the adjusted bearing at B =. Press R/S and the adjusted distance will be displayed at D =. Repeat pressing of R/S will display adjusted bearings and distances.

After the last adjusted line, a final R/S will cause the calculator to display the adjusted area at $A = (The area = -357,597.8300 \text{ m}^2 \text{ but is meaningless since this is not a closed polygon})$

LINE	STEP	STEP X Y		Z	Т
A001	LBL A				
A002	CLVARS		START NEW	ADJUSTMENT	
A003	$CL\Sigma$				
A004	-1				
A005	STO I				
A006	2		START NEW	LINE OF FI	GURE
A007	STO+I	Increment	indirect s	torage reg	isters
800A	STO+J				
A009	0				
A010	STO B				
A011	STO D				
A012	INPUT B	Enter Bear	ring (D.MMS	S)	
A013	HMS→				
A014	STO B				
A015	INPUT D	Enter Dist	cance d_k		
A016	STO+L	Accumulate	e distances	1	
A017	RCL B	Brg	Dist		
A018	+	Brg+Dist			
A019	x = 0 ?				
A020	GTO A044	Yes! End c	of Data; GO	TO adjustr	ment
A021	RCL B	Brg			
A022	RCL D	Dist	Brg		
A023	XEQ Z015	ΔN_k	$\Delta E_{\rm k}$		
A024	Σ +	n	$\Delta E_{\rm k}$		
A025	LASTX	ΔN_k	ΔE_{k}		
A026	STO(J)				
A027	x <> y	ΔE_k	ΔN_k		
A028	STO(I)	11	11		
A029	×	$\Delta E_k \Delta N_k$			
A030	RCL D	d_k	$\Delta E_k \Delta N_k$		
A031	÷	$\Delta E_k \Delta N_k / d_k$			
A032	STO+S				
A033	RCL(I)	$\Delta E_{\rm k}$			
A034	x^2	(Δ E $_{ m k}$) 2			
A035	RCL D	dk			
A036	÷	(Δ E $_{ m k}$) 2 /d $_{ m k}$			
A037	STO+R				
A038	RCL(J)	ΔN_k			
A039	x ²	(ΔN_k) ²			
A040	RCL D	dk			
A041	÷	(ΔN_k) ² / d_k			

LINE	STEP	X	Y	Z	Т
A042	STO+V				
A043	GTO A006	GO FOR nex	kt line		
A044	INPUT X	Enter D_E			
A045	INPUT Y	Enter D_N			
A046	INPUT F	Enter Flag	g (Bowditch	= 1; Crano	dall = 2)
A047	RCL F	Flag			
A048	1	1	Flag		
A049	x = y?				
A050	GTO A056	Yes! GO TO) Bowditch	adjustment	
A051	RCL F	Flag			
A052	2	1	Flag		
A053	x = y?				
A054	GTO A079	Yes! GO TO	Crandall	adjustment	
A055	GTO A046				
A056	XEQ A128	D _N -S _N	$D_E - S_E$	BOWDITCH A	DJUSTMENT
A057	RCL L	L	D _N -S _N	$D_E - S_E$	
A058	÷	(D _N -S _N)/L	$D_E - S_E$		
A059	STO Y				
A060	x <> y	$D_E - S_E$	(D _N -S _N)/L		
A061	RCL L	L	$D_E - S_E$	$(D_N - S_N) / L$	
A062	÷	$(D_E - S_E) / L$	$(D_N - S_N) / L$		
A063	STO X				
A064	XEO A135	Set regist	ers C,I,J,	A	
	~	Increment	counters f	or next li	ne of
A065	XEQ A144	adjusted f	igure		
A066	XEQ A151	Get UNADJU	JSTED Beari	ng and Dist	tance
A067	RCL D	dk			
A068	RCL X	(D _E -S _E)/L	d _k		
A069	×	$dE_k = d_k [(D_k)]$	$_{\rm E} - S_{\rm E}) / L] =$	correction	to $\Delta \mathtt{E}_{k}$
A070	STO+(I)				
A071	RCL D	d _k			
A072	RCL Y	(D _N -S _N)/L	d _k		
A073	×	$dN_{k} = d_{k} [(D)$	$(N - S_N) / L] =$	correction	to AN _k
A074	STO+(J)				
A075	XEO A162	Compute Ar	rea contrib	ution of Al	DJ. line
A076	xeo A151	Get ADJUSI	ED Bearing	and Dista	nce
A077	xeo A177	View Adius	sted Bearin	q and Dista	ance
A078	GTO A065	GO FOR nex	t line of	fiqure	
A079	RCL R	$a = \Sigma [(\Lambda E_{\nu})]$	$()^{2}/d_{r}$	CRANDALL A	DJUSTMENT
A080	RCL V	$b = \Sigma \left[\left(\Lambda N \right) \right]$	$(d_{1})^{2}/d_{1}$	$a = \Sigma \left[(\Lambda E_{1}) \right]$	$\frac{1}{2} \frac{d_{\rm b}}{d_{\rm b}}$
A081	x	ab	, ,		, , <u> </u>
A082	RCI S	$\alpha = \sum \left[\Lambda \mathbf{E} \right] \Lambda$	N,) / d. 1	ab	
7002		$C - \Delta L \Delta E_k \Delta$	$[n_k]/(u_k)$		
AUDJ	0 016	C	au	1	

SHEET 2 OF 6 SHEETS

LINE	STEP	TEP X Y		Z	Т
A084	x^2	c ² ab			
A085	-	ab-c ²			
A086	STO T				
A087	XEQ A128	D _N -S _N	$D_E - S_E$		
A088	STO×S				
A089	STO×R				
A090	<i>x</i> <> <i>y</i>	$D_E - S_E$	$D_N - S_N$		
A091	STO×V				
A092	STO×U				
A093	RCL R	$a(D_N-S_N)$			
A094	RCL U	$C(D_E - S_E)$	$a(D_N-S_N)$		
A095	-	a(D _N -S _N)-c	$(D_E - S_E)$	•	
A096	RCL T	ab-c ²	$a(D_N-S_N)-C$	$(D_E - S_E)$	
A097	÷	$k_2 = [a (D_N - S_1)]$	$(D_E - S_E)$]/(ab-c ²)	
A098	STO W				
A099	RCL V	$b(D_E - S_E)$			
A100	RCL S	$C(D_N - S_N)$	$b(D_E - S_E)$		
A101	-	$b(D_E - S_E) - c$	(D _N -S _N)		
A102	RCT T	ab-c ²	$b(D_E - S_E) - C$	(D _N -S _N)	
A103	÷	$k_1 = [b(D_E - S_E)]$	$_{\rm E}$) - C ($\rm D_{\rm N}$ - S $_{\rm N}$)]/(ab-c ²)	
A104	STO T				
A105	XEQ A135	Set regist	cers C,I,J,	А	
A106	XEQ A144	Increment adjusted f	counters f Tigure	or next lin	ne of
A107	XEQ A151	Get UNADJU	JSTED Beari	ng and Dist	tance
A108	RCL(I)	$\Delta E_{\rm k}$			
A109	RCL T	k ₁	$\Delta E_{\rm k}$		
A110	×	$k_1 \Delta E_k$			
A111	RCL(J)	ΔN_k	$k_1 \Delta E_k$		
A112	RCL W	k ₂	ΔN_k	$k_1 \Delta E_k$	
A113	×	$k_2 \Delta N_k$	$k_1 \Delta E_k$		
A114	+	$v_k = k_1 \Delta E_k + k_2$	ΔN_k		
A115	STO+D				
A116	RCL B				
A117	HMS→	Brg			
A118	RCL D	Dist	Brg		
A119	XEQ Z015	ΔN_k	$\Delta E_{\rm k}$		
A120	STO(J)				
A121	<i>x</i> <> <i>y</i>	ΔE_{k}	ΔN_k		
A122	STO(I)				
A123	XEQ A162	Compute Ar	rea contrib	ution of Al	DJ. line
A124	XEQ A177	View Adjus	sted Bearin	g and Dista	ance
A125	GTO A106	GO FOR nex	ct line of	figure	
A126	VIEW A	Area			

LINE	STEP	X	Y	Z	Т
A127	GTO A002	GO FOR new	v figure to	adjust	
A128	RCL X	D _E			
A129	Σy	$S_{E} = \Sigma \Delta E_{k}$	D _E		
A130	-	$D_E - S_E$			
A131	RCL Y	D _N	$D_E - S_E$		
A132	Σx	$S_{N} = \Sigma \Delta N_{k}$	D _N	$D_E - S_E$	
A133	-	D _N -S _N	$D_E - S_E$		
A134	RTN				
A135	n	n	SET REGIST	ERS C,I,J,	A
A136	STO C	count = n			
A137	CLΣ				
A138	-1	-1			
A139	STO I				
A140	0	0			
A141	STO J				
A142	STO A				
A143	RTN				
A144	2	2	INCREMENT	REGISTERS	I,J
A145	STO+I	Increment	indirect s	torage reg.	. for $\varDelta E$
A146	STO+J	Increment	indirect s	torage reg.	. for ΔN
A147	RCL C	count			
A148	x = 0 ?				
A149	GTO A126	Yes! GO FC	DR Area of	adjusted f	igure
A150	RTN				
A151	360	360	BEARING &	DISTANCE S	UBROUTINE
A152	RCL(I)	ΔE_k	360		
A153	RCL(J)	ΔN_k	$\Delta E_{\rm k}$	360	
A154	XEQ Z002	d _k	Brg _k	360	
A155	STO D				
A156	R↓	Brg _k	360		
A157	<i>x</i> < 0 ?				
A158	+				
A159	→HMS	Brg(D.MMSS	5)		
A160	STO B				
A161	RTN				
A162	RCL(I)	$\Delta E_{\rm k}$	AREA SUBRO	UTINE	
A163	RCL(J)	ΔN_k	ΔE_k		
A164	Σ +	n	ΛE _l		
A165	R↓	ΔE_k			
A166	LASTX	ΔN_k	ΔE_{k}		
A167	Σy	$\Sigma \Delta E_k$	ΔN_k	ΔE_{k}	
A168	×	$\Delta N_k \Sigma \Delta E_k$	$\Delta E_{\rm k}$		
A169	x <> v	ΔE_{k}	$\Delta N_k \Sigma \Delta E_k$		
		17			

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HP35s PROGRAM SHEET

ADJUSTMENT PROGRAM

LINE	STEP	X	Y	Z	Т
A171	×	$\Delta \mathtt{E}_{k} \Sigma \Delta \mathtt{N}_{k}$	$\Delta N_k \Sigma \Delta E_k$		
A172	-	$\Delta N_k \Sigma \Delta E_k - \Delta E_k$	$_{\rm k}\Sigma\Delta{ m N}_{\rm k}$		
A173	2				
A174	÷	Area compo	onent		
A175	STO+A	Accumulate	e area		
A176	RTN				
A177	VIEW B	(Adjusted)	Bearing (D.MMSS)	
A178	VIEW D	Adjusted L	Distance		
A179	1	1			
A180	STO-C	Decrement	count		
A181	RTN				

STORAGE REGISTERS

Α	Area
В	Bearing
С	count = counter for lines of figure $0 \le \text{count} \le n$
D	Distance d_k ; d_k+v_k
I	Indirect storage register for $\Delta extsf{E}$
J	Indirect storage register for $\Delta extsf{N}$
L	Cumulative distance L = Σd_k
R	$a = \Sigma [(\Delta E_k)^2 / d_k] ; a (D_N - S_N)$
S	$c = \Sigma \left[\Delta E_{k} \Delta N_{k} \right) / d_{k}] ; c \left(D_{N} - S_{N} \right)$
Т	$ab-c^{2}$; $k_{1}=[b(D_{E}-S_{E})-c(D_{N}-S_{N})]/(ab-c^{2})$
U	$C i C (D_E - S_E)$
V	$b = \Sigma [(\Delta N_k)^2 / d_k] ; b (D_E - S_E)$
W	$k_2 = [a(D_N - S_N) - c(D_E - S_E)] / (ab - c^2)$
X	$D_E = E_{END} - E_{START} ; (D_E - S_E) / L$
Y	$D_N = N_{END} - N_{START}$; $(D_N - S_N) / L$

PROGRAM LENGTH AND CHECKSUM

LN = 558; CK = 68A4

★ Length & Checksum: $\square \square \square 2$; $\square \square \square$ (Hold)

SHEET 5 OF 6 SHEETS

PROGRAM NOTES

Lines	A021	to	A042	convert bearings and distances of the figure to east and north components $(\Delta E_k, \Delta N_k)$ and form the sums: $S_E = \Sigma y = \Sigma \Delta E_k;$ $S_N = \Sigma x = \Sigma \Delta N_k;$ $a = \Sigma [(\Delta E_k)^2/d_k];$ $b = \Sigma [(\Delta N_k)^2/d_k];$
Lines	A056	to	A078	Bowditch adjustment using subroutines
Lines	A079	to	A125	Crandall adjustment using subroutines A128, A135, A144, A151, A162, A177.
Lines	A128	to	A134	is a subroutine that computes the misclosures east and north: D_E-S_E and D_N-S_N where $D_E = E_{END}-E_{START}$ and $D_N = N_{END}-N_{START}$ Note that for a loop traverse (or closed figure) $D_E = D_N = 0$
Lines	A135	to	A143	is a subroutine that sets storage registers C.I.J.A and clears Σ
Lines	A144	to	A150	is a subroutine that increments indirect storage registers I,J and tests to see if count=0. If count=0 then all adjusted bearings and distances have been computed and the area will be displayed.
Lines	A151	to	A161	is a subroutine that computes bearings
Lines Lines	A162 A177	to to	A176 A181	is an area subroutine. is a subroutine for displaying Bearing (D.MMSS) and Distance and decrementing the counter.

The calculator must contain LBL Z which contains the Polar to Rectangular routines.

XEQ Z002 on line A154 is the Rectangular \rightarrow Polar conversion XEQ Z015 on lines A023,A119 is the Polar \rightarrow Rectangular conversion.