Help message for MATLAB function refval.m

>> help refval
Function refval determines the geometric and physical constants of the
equipotential rotational reference ellipsoid of the Geodetic Reference
System 1980 (GRS80) defined by:

- \(a\) - semi-major axis,
- \(GM\) - geocentric gravitational constant of the Earth,
- \(J2\) - dynamical form factor and
- \(\omega\) - angular velocity of the Earth

For GRS80 the defining parameters are

\[
\begin{align*}
a & = 6378137 \text{ m} \\
GM & = 3.986005 \times 10^{14} \text{ m}^3/\text{s}^2 \\
J2 & = 1.082630 \times 10^{-3} \\
\omega & = 7.292115 \times 10^{-5} \text{ rad/s}
\end{align*}
\]

Output from MATLAB function refval.m

>> refval

GEODETIC REFERENCE SYSTEM 1980
==================================
Defining Constants (exact)

\[
\begin{align*}
a & = 6378137.000 \text{ m} \\
GM & = 3.986005 \times 10^{14} \text{ m}^3/\text{s}^2 \\
J2 & = 1.082630 \times 10^{-3} \\
\omega & = 7.292115 \times 10^{-5} \text{ rad/s}
\end{align*}
\]

Derived Geometrical Constants of Normal Ellipsoid

\[
\begin{align*}
b & = 6356752.314140 \text{ m} \\
E & = 521854.009700 \text{ m} \\
c & = 6399593.625864 \text{ m} \\
e^2 & = 6.694380022903e-03 \\
e^2_p & = 6.739496775482e-03 \\
f & = 3.352810681184e-03 \\
flat & = 298.257222100882 \\
n & = 1.679220394629e-03 \\
Q & = 10001965.729230 \text{ m} \\
R1 & = 6371008.771380 \text{ m} \\
R2 & = 6371007.180884 \text{ m} \\
R3 & = 6371000.789974 \text{ m}
\end{align*}
\]

Derived Physical Constants of Normal Ellipsoid

\[
\begin{align*}
U_0 & = 62636860.850046 \text{ m}^2/\text{s}^2 \\
J4 & = -2.370912218650e-06 \\
J6 & = 6.083470628388e-09 \\
J8 & = -1.142681405971e-11 \\
m & = 3.449786003078e-03 \\
g_e & = 9.780326771534 \text{ m/s}^2 \\
g_p & = 9.832186368521 \text{ m/s}^2 \\
f^* & = 0.005302440113 \\
k & = 0.001931831453
\end{align*}
\]

>>
function refval
% Function refval determines the geometric and physical constants of the
equipotential rotational reference ellipsoid of the Geodetic Reference
% System 1980 (GRS80) defined by:
% a - semi-major axis,
% GM - geocentric gravitational constant of the Earth,
% J2 - dynamical form factor and
% omega - angular velocity of the Earth
% For GRS80 the defining parameters are
% a = 6378137 m
% GM = 3.986005e+14 m^3/s^2
% J2 = 1.08263e-03
% omega = 7.292115e-05 rad/s

%==========================================================================

% Function: refval
% Useage:   refval;
% Author:   Rod Deakin,
% Department of Mathematical and Geospatial Sciences,
% RMIT University,
% GPO Box 2476V, MELBOURNE VIC 3001
% AUSTRALIA
% email: rod.deakin@rmit.edu.au
% Date:     Version 2   21 May 2014
% Note that Version 1 was written as a C++ program in June 1996 with
% revisions in December 1996 and June 1997.
% Functions Required:
% None
% Remarks:
% Function refval() determines the geometric and physical constants of the
equipotential rotational reference ellipsoid defined by:
% a - semi-major axis,
% GM - geocentric gravitational constant of the Earth,
% J2 - dynamical form factor and
% omega - angular velocity of the Earth.
% Given the four defining parameters above, geometric and physical
% constants can be derived by methods and formulae set out in Ref[1].
% Theory is given in Ref[2] and Ref[4] and a FORTRAN program listing is
given in Ref[3].
% e2 (first eccentricity squared) is the fundamental derived constant from
% which the other geometric constants can be computed. Ref[1] and Ref[4]
% show how e2 is linked to a, GM, J2 and omega via the formulae
% e2 = 3*J2 + (omega^2)*(a^3)/GM * 4/15*(e^3)/2q0
% This equation must be solved iteratively since e appears on both sides
% of the equal sign. A summation formula for 2q0/(e^3) can be derived by
% considering the equations given in Ref[1] p.398. Also see ref[4],
eq(73), p.28.
% The following geometric constants of the ellipsoid are evaluated
% b - semi-minor axis of ellipsoid (metres)
% c - polar radius of curvature (metres)
% ep2 - second eccentricity squared: ep = E/b
% E - linear eccentricity: E = sqrt(a^2 - b^2) = a*e
% f - flattening of ellipsoid
% flat - denominator of flattening: f = 1/flat
% n - third flattening of ellipsoid: n = f/(2-f)
% Q - quadrant distance of ellipsoid (metres)
% R1 - radius of sphere having mean radius: R1 = (2*a + b)/3
% R2 - radius of sphere having same surface area of ellipsoid
% R3 - radius of sphere having same volume of ellipsoid
The following physical constants of the ellipsoid are evaluated

- \( \gamma_e \) - normal gravity at equator (m/s^2)
- \( \gamma_p \) - normal gravity at pole (m/s^2)
- \( J_4, J_6, J_8, \ldots \) - coefficients of Legendre polynomials in the spherical harmonic expansion of the gravitational potential \( V \).
- \( k \) - constant in Pizetti's equation for normal gravity.
- \( m \) - a derived physical constant: \( m = \omega^2 a^2 b/G \)
- \( q_0 \) - \( q \) (subscript zero), physical constant used in computation of \( \gamma_e, \gamma_p \)
- \( qp_0 \) - \( q \)-primed(subscript zero), physical constant used in computation of \( \gamma_e, \gamma_p \)
- \( U_0 \) - normal gravity potential at ellipsoid (m^2/s^2)

**Variables:**

- \( a \) - semi-major axis of ellipsoid (metres)
- \( b \) - semi-minor axis of ellipsoid (metres)
- \( c \) - polar radius of curvature (metres)
- \( e \) - (first) eccentricity of ellipsoid: \( e = E/a \)
- \( e_2 \) - (first) eccentricity squared
- \( e_2^l \) - approximate value of \( e_2 \)
- \( e_2^p \) - second eccentricity squared
- \( E \) - linear eccentricity: \( E = \sqrt{a^2 - b^2} = a \cdot e \)
- \( f \) - flattening of ellipsoid: \( f = (a-b)/a = 1-\sqrt{1-e^2} \)
- \( f_1 \) - gravity flattening
- \( flat \) - denominator of flattening: \( f = 1/flat \)
- \( \gamma_e \) - normal gravity at equator (m/s^2)
- \( \gamma_p \) - normal gravity at pole (m/s^2)
- \( GM \) - geocentric gravitational constant (m^3/s^2)
- \( i, j \) - integer counters
- \( Jvec \) - vector of zonal harmonic terms for computation of normal gravitational potential \( V \)
- \( J_2 \) - dynamical form factor
- \( k \) - constant in Pizetti's equation for normal gravity.
- \( m \) - a derived physical constant: \( m = \omega^2 a^2 b/G \)
- \( ml \) - a derived physical constant: \( ml = \omega^2 a^3/G \)
- \( n \) - third flattening of ellipsoid: \( n = f/(2-f) \)
- \( n_2, n_4, \ldots \) - even powers of \( n \)
- \( N \) - size of vector \( Jvec \)
- \( \omega \) - angular velocity of earth (radians/sec)
- \( q_0 \) - \( q \) (subscript zero), physical constant used in computation of \( \gamma_e, \gamma_p \)
- \( qp_0 \) - \( q \)-primed(subscript zero), physical constant used in computation of \( \gamma_e, \gamma_p \)
- \( Q \) - quadrant distance of ellipsoid (metres)
- \( R_1 \) - radius of sphere having mean radius: \( R_1 = (2a + b)/3 \)
- \( R_2 \) - radius of sphere having same surface area as ellipsoid
- \( R_3 \) - radius of sphere having same volume as ellipsoid
- \( sgn \) - sign function: \( sgn = 1 \) or \( sgn = -1 \)
- \( U_0 \) - normal gravity potential at ellipsoid (m^2/s^2)
- \( x \) - local variable

**References:**

% set defining constants of rotational equipotential ellipsoid for GRS80

%-----------------------------------------------------------------------
a = 6378137.0;
GM = 3.986005e+14;
J2 = 1.08263e-03;
omega = 7.292115e-05;

%-----------------------------------------------------------------------

% compute e2 (first eccentricity squared)
% See Ref[1], p.398
% set initial value of first eccentricity squared
m1 = omega^2*a^3/GM;
e21 = 3*J2 + m1;
% set e2 = initial value
e2 = e21;
for i = 1:10
    ep2 = e2/(1-e2);
x = (1/(1-e2))^(3/2);
sgn = 1;
twoq0 = 0;
% this loop calculates 2q0/(e^3)
for j = 1:20
    x = x*ep2;
    twoq0 = twoq0 + (sgn*(4*j/(2*j+1)/(2*j+3)*x));
    sgn = -sgn;
end
% set e2 = initial value + correction
e2 = e21 + (m1*((4/15/twoq0)-1));
end
% e2 now known, calculate geometric constants of ellipsoid

%-------------------------------------------------------
% compute GEOMETRIC constants of equipotential ellipsoid
%-------------------------------------------------------

% This section computes geometric constants of the equipotential ellipsoid.
% The constants include:
% e: eccentricity of ellipsoid
% ep2: 2nd eccentricity squared
% ep: 2nd eccentricity
% f: flattening
% flat: denominator of flattening
% b: semi-minor axis of ellipsoid
% a: semi-major axis of ellipsoid
% n: 3rd flattening
% c0: quadrant distance
% R1: radius of equivalent sphere at equator
% R2: radius of equivalent sphere at pole
% R3: radius of equivalent sphere at any point

% computation of quadrant length of ellipsoid. See Ref[5], eq (39).
% n2 = n^n; % even powers of n
% n4 = n^4n2; % n6 = n^6n2; % n8 = n^8n2; % c0 = a/(1+n)*c0*(pi/2); % quadrant distance
% computation of radii of equivalent spheres. See Ref[6], pp. 86-87.
% R1 = (2*a+b)/3;
% R2 = sqrt(a^2/2*(1+(1-e2)/(2*e)*log((1+e)/(1-e)))));
% R3 = (a^2*b)^(1/3);

%-------------------------------------------------------
% compute PHYSICAL constants of equipotential ellipsoid
%-------------------------------------------------------

% compute normal potential of the reference ellipsoid U0
U0 = GM/E*atan(ep) + omega^2*a^2/3;
% compute normal gravity at equator and poles
% see also Ref[1] p.398
% see also Ref[1] p.400; Ref[4] eq.(52)
compute normal gravity at the equator and the pole
% formulae for gamma_e and gamma_p given in Ref[2] eqs (2-73) and (2-74),
% p.69; Ref[1] p.400 and Ref[4], eqs (62a), (62b).
m = m1/a*b;
gamma_e = GM/a/b*(1-m-(m/6*ep*qp0/q0));
gamma_p = GM/a*a*(1+(m/3*ep*qp0/q0));
% compute gravity flattening
f1 = gamma_p/gamma_e - 1;
% compute constant k in Pizzetti's equation for normal gravity
k = b*gamma_p/(a*gamma_e) - 1;

%---compute the coefficients J4, J6, J8, ... J20
% set dimensions of vector Jvec()
N = 20;
Jvec = zeros(N,1);
% This vector is used to store the coeff's J2, J4, J6, ... J20 where J(2n) are
% related to J2. See Ref[2], p.73, eqs. (2-92), (2-92') and Ref[4] eq.(86)
% Also Jvec(1,1) = Jvec(3,1) = Jvec(5,1) = ... = Jvec(odd,1) = 0.
% The elements of Jvec() are used in computations of the normal potential V
% and its derivatives. Note also that the elements of Jvec() are zonal
% terms, i.e. the order is zero (m=0) thus they are also equal to quasi-
% normalised coefficients. See Ref[3], eq. 21, p. 258.

% set value of Jvec(2,1)
Jvec(2,1) = J2;
% set even values of Jvec() */
for j=2:N/2
    sgn = (-1)^+(j+1);
    x1 = 3/(2*j+1)/(2*j+3)*e2^j;
    x2 = 1 - j + (5*j*J2/e2);
    Jvec(2*j,1) = sgn * x1 * x2;
end

% print headings and computed data
fprintf('

GEODETIC REFERENCE SYSTEM 1980');
fprintf('
==============================');
fprintf('
Defining Constants (exact) 
');
fprintf('
    a = %12.3f m          semi-major axis',a);
fprintf('
   GM = %9.6e m^3/s^2    geocentric gravitational constant',GM);
fprintf('
   J2 = %9.6e            dynamical form factor',J2);
fprintf('
omega = %9.6e rad/s      angular velocity',omega);

Derived Geometrical Constants of Normal Ellipsoid 
');
fprintf('
    b = %15.6f m       semi-minor axis',b);
fprintf('
    E = %15.6f m       linear eccentricity',E);
fprintf('
    c = %15.6f m       polar radius of curvature',c);
fprintf('
    ep2 = %15.12e      eccentricity squared',ep2);
fprintf('
    f = %15.12e      flattening',f);
fprintf('
    flat = %15.12f      denominator of flattening',flat);
fprintf('
    n = %15.12e      3rd flattening',n);
fprintf('
    Q = %15.6f m       quadrant distance',Q);
fprintf('
    R1 = %15.6f m       mean radius R1 = (2a+b)/3',R1);
fprintf('
    R2 = %15.6f m       radius of sphere of same surface area',R2);
fprintf('
    R3 = %15.6f m       radius of sphere of same volume',R3);

Derived Physical Constants of Normal Ellipsoid 
');
fprintf('
    U0 = %15.6f m^2/s^2 normal gravity potential',U0);
fprintf('
    J4 = %15.12e spherical-harmonic coefficient',Jvec(4));
fprintf('
    J6 = %15.12e spherical-harmonic coefficient',Jvec(6));
fprintf('
    J8 = %15.12e spherical-harmonic coefficient',Jvec(8));
fprintf('
    m = % 15.12e',m);
fprintf('
    g_e = %15.12f m/s^2 normal gravity at equator',gamma_e);
fprintf('
    g_p = %15.12f m/s^2 normal gravity at pole',gamma_p);
fprintf('
    f* = %15.12f gravity flattening',f1);
fprintf('
    k = %15.12f constant in Pizzetti''s equation',k);

');