STREAM DISCHARGE MEASUREMENT (STREAM GAUGING)

The discharge Q from creeks, small streams and rivers can be determined by a number of methods, four of which are discussed below. The determination of stream discharges is known as gauging and the choice of a particular method is one for the hydrologist or engineer. The four methods of stream gauging to be considered are (i) the *Area Velocity Method,* (ii) the *Slope Area Method*, (iii) the *Weir Method* and (iv) the *Volumetric Method.*

AREA VELOCITY METHOD

This method measures the volume of water passing a selected stream cross section in a certain time interval. If the cross section area A is known and the mean velocity of the stream flow V determined, the discharge Q is given by

$$Q = AV \tag{1}$$

where

Q is the discharge expressed in cubic metres per second (m^3/s) or cumecs
A is the stream cross section area (m^2)
V is the mean velocity of stream flow (m/s)

The stream line adjacent to the selected cross section should be straight and of uniform slope, free from obstructions and allowing a regular non-turbulent flow. The stream cross section shape can be determined by level and staff, tacheometry (Total Station), soundings from a cross-wire or similar means and the cross section area A calculated by Simpson's Rule or the Trapezoidal Rule. The mean velocity V of the stream flow at the cross section can be determined by timed observation of floats or by current meters or other practical methods, but it is important to understand the velocity distribution of flowing water.

Stream Velocity Distribution over a Cross Section

The velocity of a river, stream or channel varies throughout the cross section and depends upon such things as the shape of the cross section, roughness of the stream bed and depth of water. Minimum velocity occurs at the bed (surface friction) and maximum velocity a little below the surface well clear of the banks in deepest water. In fast flowing streams the position of maximum velocity moves from bank to bank. Figure 1 shows a stream cross section with the dotted curves representing lines of equal velocity. In regular shaped channels (rectangular, trapezoidal and circular), the Velocity–Depth curve is approximately parabolic with maximum velocity situated below the surface at 0.2 to 0.3 depth.

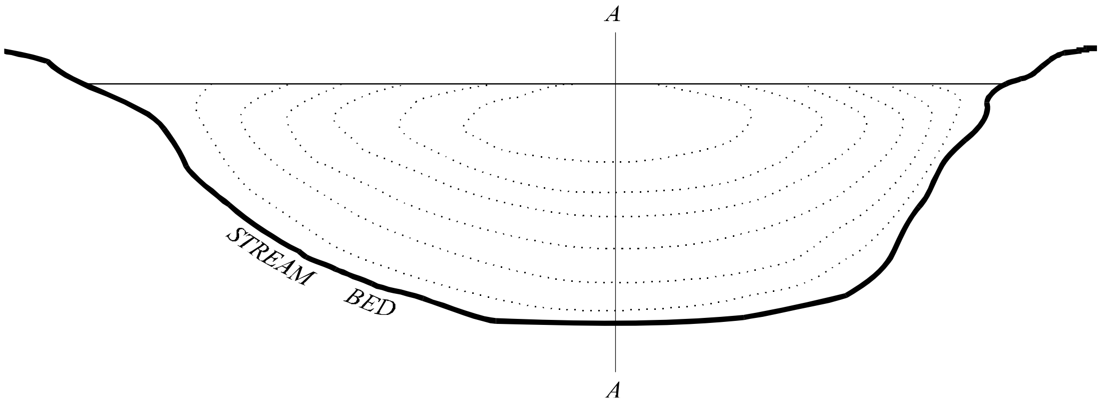

Figure 1 – Stream Cross-Section and Velocity distribution

Figure 2 shows a parabolic Velocity–Depth curve along the section *AA*. The equation of the curve takes the form $v = ad^2 + bd + c$ where v is velocity, d is depth, a and b are coefficients and c is a constant. For depth d equal to one unit, velocity v at the surface equal to one unit and maximum velocity $v_{\max} = 0.3d$ the velocity is given by $v = -\frac{5}{2}d^2 + \frac{3}{2}d + 1$. Using the *mean-value theorem* of integral calculus $\left[\bar{x} = \frac{1}{b-a}\int_a^b f(x)\, dx \right]$ gives the mean velocity $\bar{v} = 0.92$, which occurs at approximately $0.65d$. The mean velocity is also very close to the average of the velocities at 0.2*d* and 0.8*d*. These figures agree closely with those given in Daugherty & Ingersoll (1954, p. 242) derived from actual river measurements.

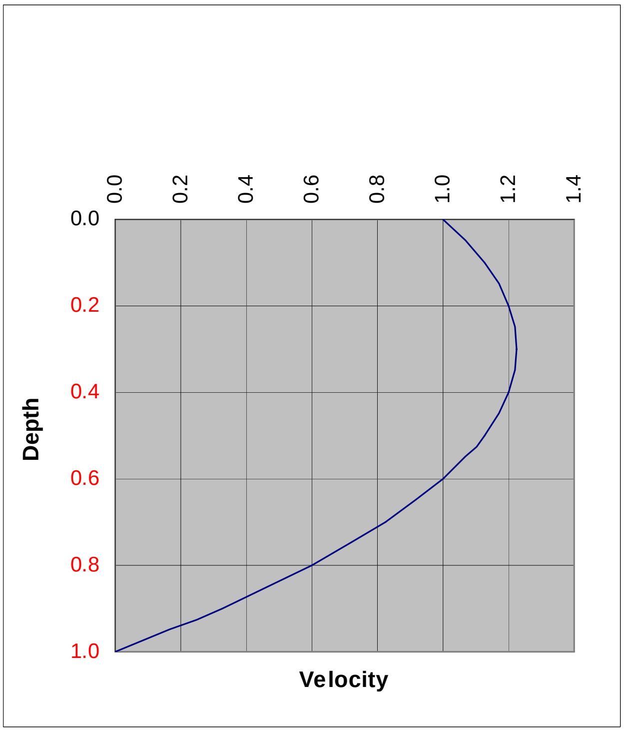

Figure 2 – Velocity versus Depth at Section AA

It is important to understand something of the nature of stream velocity distribution to properly position flow monitoring equipment.

Determination of Stream Velocity

1. PITOT TUBE. The Pitot tube is a simple device used for measuring velocities of stream flow named after Henri Pitot (see citation from Encyclopaedia Britannica below) who used a bent glass tube in 1730 to measure velocities in the River Seine. The Pitot Tube is illustrated in Figure 3. One end is pointed directly into the stream flow at depth H and the water rises up the tube until all its kinetic energy is converted into potential energy; a height h above the free surface of the stream. The fluid streamlines divide as they approach the blunt end of the tube and at A there is complete stagnation, since the fluid at this point is moving neither up nor down nor to the right or left. It follows from Bernoulli's law (to be discussed later) that

$$v = \sqrt{2gh} \tag{2}$$

where

v	is the velocity of the stream flow (m/s)
g	is the acceleration due to gravity (m/s^2)
h	is the velocity head (m)

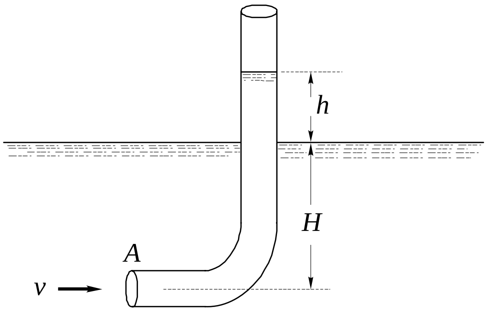

Figure 3 – Pitot tube

Pitot, Henry (b. May 3, 1695, Aramon, Fr. – d. Dec. 27, 1771, Aramon), French hydraulic engineer and inventor of the Pitot tube, which measures flow velocity.
Beginning his career as a mathematician and astronomer, Pitot won election to the Academy of Sciences in 1724. He became interested in the problem of flow of water in rivers and canals and discovered that much contemporary theory was erroneous – for example, the idea that the velocity of flowing water increased with depth. He devised a tube, with an opening facing the flow, that provided a convenient and reasonably accurate measurement of flow velocity and that has found wide application ever since (*e.g.,* in anemometers for measuring wind speed). Appointed chief engineer for Languedoc, he performed a variety of maintenance and construction works on canals, bridges, and drainage projects. His major work was construction of an aqueduct for the city of Montpellier (1753-86), including a stone arch Roman-type section a kilometre (more than 1/2 mile) in length.

Measurements of the velocity head h at various depths H can be used to determine average stream velocity.

2. FLOATS. A simple way of measuring the velocity of flow is by means of floats. The surface velocity at any section may be measured by a single *surface float.* The time traversed over a measured distance yields the velocity, but velocity may be affected by wind and air resistance. A better method is to use a *rod float*, being a wooden rod (or hollow tube) weighted at the end to float vertically in the stream. The rod should be as long as the depth of the stream and will travel at the mean velocity of the section (Lewitt 1961, p. 265).

3. CURRENT METER. A schematic diagram of a type of current meter is shown in Figure 4. It consists of wheel containing blades or cups, which are rotated by the flowing water; these are headed towards the current by means of a tail on which vanes or fins are attached. An electric current is passed to the wheel from a battery above the water by means of wires and a commutator is fixed to the shaft of the revolving blades, which makes and breaks the electric circuit each revolution. The revolutions are recorded by a counter above the water, which is worked by the electric current. The meter is lowered into the water to the required depth and the velocity obtained from the revolution counter (Lewitt 1961, p. 263).

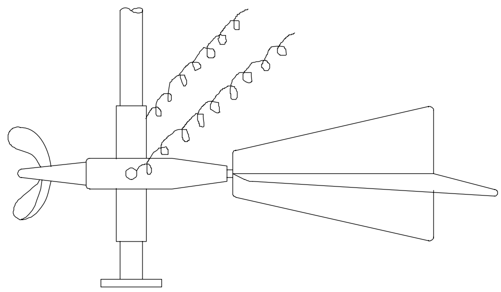

Figure 4 – Current Meter

A current meter must be calibrated, so the revolutions per second can be converted into velocity. This can be done in the field by suspending the meter from a boat and timing runs over measured distances in still water. Alternatively, a current meter may be calibrated in a laboratory by towing the meter from a trolley in a testing tank at uniform speeds. In either case, a rating curve (usually a straight line) can be established linking revolutions per second to velocity.

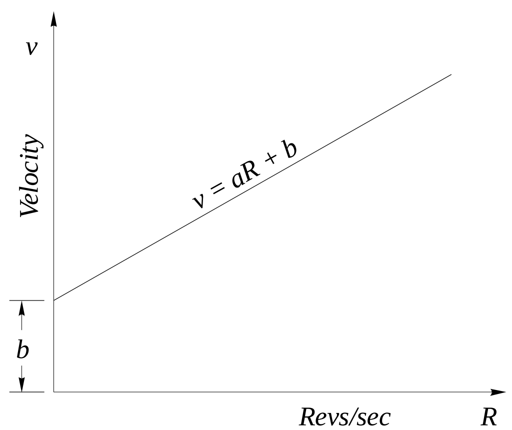

Figure 5 – Rating Curve (straight line) for Current Meter

Figure 5 shows a rating curve (straight line) for a current meter. For any velocity $v = aR + b$ where v is velocity of the stream flow (m/s), R is the number of revolutions per second and a and b are constants for the particular current meter determined from the calibration measurements. The velocity at zero revolutions is the amount required to overcome friction in the current meter.

Current meter readings are only necessary in one cross section. The section is divided into partial areas (trapezoids and triangles against banks) with the meter in mid or centroidal vertical for each partial area. Meter readings can be taken by wading (shallow water), from endless wire rigs, by suspending from bridges, or from boats. Whichever method is adopted, the meter must be held at the required depth and the revolutions counted for the desired period. Meter readings can be:

(a) Single observations at the surface or at the depth of mean velocity. Surface velocities may be read in swift currents or streams in flood, when it would be difficult to maintain the meter at any given depth. In such cases the meter may be held at a depth just below the surface (clear of wave action) and the velocity multiplied by 0.9 to give the mean velocity. Under quieter conditions the mean velocity may be measured directly at 0.6 of the depth.

(b) Multiple observations. In the simplest case, the mean velocity may be determined from meter readings at 0.2 and 0.8 of the depth. A more accurate method is to take readings at regular intervals of the stream depth. The mean velocity is obtained from a plot of the Velocity-Depth curve (see Figure 2) by calculating the area between the curve and Depth axis and dividing by the depth.

$$\text{Mean Velocity} = \frac{\text{Area under curve}}{\text{Depth}}$$

This is a practical analogue of the mean value theorem of integral calculus.

SLOPE–AREA METHOD

This method differs from the *Area–Velocity Method* in that the mean velocity *V* of the stream flow is determined <u>indirectly</u> by hydraulic formulae rather than <u>directly</u> from floats, Pitot tube, or current meter readings. Several formulae have been used over the past century to compute *V*, all empirically derived from observations of actual stream (or channel) flow or simulations with models. The original hydraulic formula by Antoine de Chezy (1718-1798, see citation below) gives the velocity of stream flow as

Chezy's formula $$V = C\sqrt{RS} \qquad (3)$$

where

- V is the velocity of the stream flow (m/s)
- C is a coefficient dependent on the "roughness" of the stream bed and derived from empirical studies
- R is the *hydraulic radius* (m) and is equal to the cross section area *A* divided by the wetted surface perimeter *p* of the channel cross section, $R = A/p$.
- S is the slope of the water surface in the stream or channel, or of the energy gradient, or of the channel bottom; these lines are regarded as parallel for steady, uniform flow. If a section of channel is straight, the difference in height Δh between two cross sections, divided by the slope distance *L* between the two sections is the slope $S = \Delta h/L$.

Chezy's original formula, developed from wooden models simulating flow in channels, was "improved" by Darcy (1803-1858), Bazin (1829-1917), Ganguillet & Kutter (in a paper in 1869) and Robert Manning (1816-1897). Information about these early hydraulic engineers is given below and an interesting history of Water Engineers can be found at http://www.soe.uoguelph.ca/webfiles/wjames/homepage/Professional/Heros.html Manning's formula, originally presented in the paper "On the Flow of Water in Open Channels and Pipes" published in *Transactions of the Institution of Civil Engineers of Ireland*, 1891, is now generally adopted for the computation of velocity of flow in channels (or streams). Manning's formula is also used for computation of flow in pipes that are not flowing under pressure.

Manning's formula $$V = \frac{k}{n}R^{2/3}S^{1/2} \qquad (4)$$

where

- V is the velocity of the stream flow (m/s)
- k is a factor depending upon the system of measurement units; $k = 1$ for SI (Metric) units and $k = 1.486$ for English units.
- n is a coefficient dependent on the "roughness" of the stream bed (or channel). Tables of *n* for use in Manning's formula have been determined for different channel/stream/pipe surfaces. See Tables 1 and 2 below.
- R is the *hydraulic radius* (m) and is equal to the cross section area *A* divided by the wetted surface perimeter *p* of the channel cross section, $R = A/p$.
- S is the slope of the water surface in the stream or channel, or of the energy gradient, or of the channel bottom; these lines are regarded as parallel for steady, uniform flow. If a section of channel is straight, the difference in height Δh between two cross sections, divided by the slope distance *L* between the two sections is the slope $S = \Delta h/L$.

Tables of Manning *'n'* values (roughness coefficients) have been determined for different Australian conditions and published in journals. Tables 1 and 2 below have been extracted from *Waterway Design – A Guide to the Hydraulic Design of Bridges, Culverts and Floodways,* pp. 21-22, a 1994 publication of AUSTROADS (the national association of road transport and traffic authorities in Australia).

	MINOR STREAMS (surface width at flood less than 30m)	*n*
(a)	Fairly regular section	
	– Some grass and weeds, little or no brush	0.030–0.035
	– Dense growth of weeds, depth of flow materially greater than weed height	0.035–0.050
	– Some weeds, light brush on banks	0.035–0.050
	– Some weeds, heavy brush on banks	0.050–0.070
	NOTE: For trees within a channel, with branches submerged at high stage, increase all above values by 0.010–0.020	
(b)	Irregular sections, with pools, slight channel meander; increase values given in (a) above by 0.010–0.020	
(c)	Mountain streams, no vegetation in channel, banks usually steep, trees and brush submerged at high stage	
	– Bottom of gravel, cobbles and few boulders	0.040–0.050
	– Bottom of cobbles, with large boulders	0.050–0.070
	FLOOD PLAINS	*n*
(a)	Pasture, no brush	
	– Short grass	0.030–0.035
	– High grass	0.035–0.050
(b)	Cultivated areas	
	– No crop	0.030–0.040
	– Mature row crops	0.035–0.045
	– Mature field crops	0.040–0.050
(c)	Brush	
	– Scattered brush, heavy weeds	0.050–0.070
	– Light brush and trees	0.060–0.080
	– Medium to dense brush	0.100–0.160
(d)	Trees	
	– Clear land with tree stumps, no sprouts	0.040–0.050
	– Same as above but with heavy growth of sprouts	0.060–0.080
	– Heavy stand of timber, a few fallen trees, little undergrowth and flood stage below branches	0.100–0.120
	– Same as above, but with flood stage reaching branches	0.120–0.160
	MAJOR STREAMS (surface width at flood stage greater than 30m)	*n*
(a)	The *n*-value is less than that for minor streams of similar description, because banks offer less effective resistance	
	Regular section with no boulders or brush	0.025–0.060
	Irregular and rough section	0.035–0.100

Table 1 – Manning roughness coefficients *n* for Natural Streams
(AustRoads 1994)

	ARTIFICIAL CHANNELS	*n*
(a)	Lined channels	
	– Concrete, smooth formed	0.012
	– Bituminous concrete	0.013–0.016
(b)	Excavated, unlined channels	
	– Uniform section, short grass	0.022–0.027
	– Fairly uniform section, grass, some weeds	0.025–0.030
	– Fairly uniform section, dense weeds, deep channel	0.030–0.035
	– Fairly uniform section, cobble bottom	0.030–0.040
(c)	Channels not maintained, weeds and brush uncut	
	– Dense weeds – high as flow depth	0.080–0.120
	– Clean bottom, brush on sides	0.050–0.080
	– Dense brush, high stage of flow	0.100–0.140

Table 2 – Manning roughness coefficients *n* for Artificial Channels
(AustRoads 1994)

To estimate the stream discharge Q using the Slope-Area Method, AustRoads (1994, p. 19) recommends the following method for natural streams:

> In natural streams of irregular cross section, it is necessary to divide the water area for a particular stage height (river or stream height) into smaller, but more or less regular subsections. Each subsection, with its appropriate A and wetted surface perimeter p is assigned an appropriate retardance factor (Manning roughness coefficient n) and the discharge calculated using Manning's equation in the following form.

$$Q = \frac{AR^{2/3}S^{1/2}}{n} \tag{5}$$

where

- Q is the discharge (m^3/s)
- A is the cross section area (m^2)
- n is a coefficient dependent on the "roughness" of the stream bed (or channel); see Tables 1 and 2.
- R is the *hydraulic radius* (m) and is equal to the cross section area A divided by the wetted surface perimeter p of the channel cross section, $R = A/p$.
- S is the slope of the water surface in the stream or channel, or of the energy gradient, or of the channel bottom; these lines are regarded as parallel for steady, uniform flow. If a section of channel is straight, the difference in height Δh between two cross sections, divided by the slope distance L between the two sections is the slope $S = \Delta h/L$.

The individual discharges are then added together to give to total stream discharge.

To estimate the discharge from channels using the Slope-Area Method, equation (5) is used with an appropriate single n value.

It should be noted that the Slope-Area Method is not a satisfactory substitute for actual field (hydraulic) measurements and should be taken as an approximation. It may sometimes be usefully employed where evidence of flood stage debris remains sometime after peak flow has passed.

The Slope-Area Method may be used to compute flood discharge in the following manner (AustRoads 1994).

Figure 6 shows a diagram of a stream cross section with flood water level at 58.0 m. The usual flow is contained within the stream bed between chainages 20 and 30 m. The water area for the flood stage height is divided into smaller, but more or less rectangular, subsections, assigning an appropriate retardance factor (Manning roughness coefficient n) to each and calculating the discharge for each subsection separately using (5).

The total discharge can then be found by adding the discharges for each subsection. This process can be repeated for other stage heights and a *Stage-Discharge Rating Curve* drawn (see Figure 7).

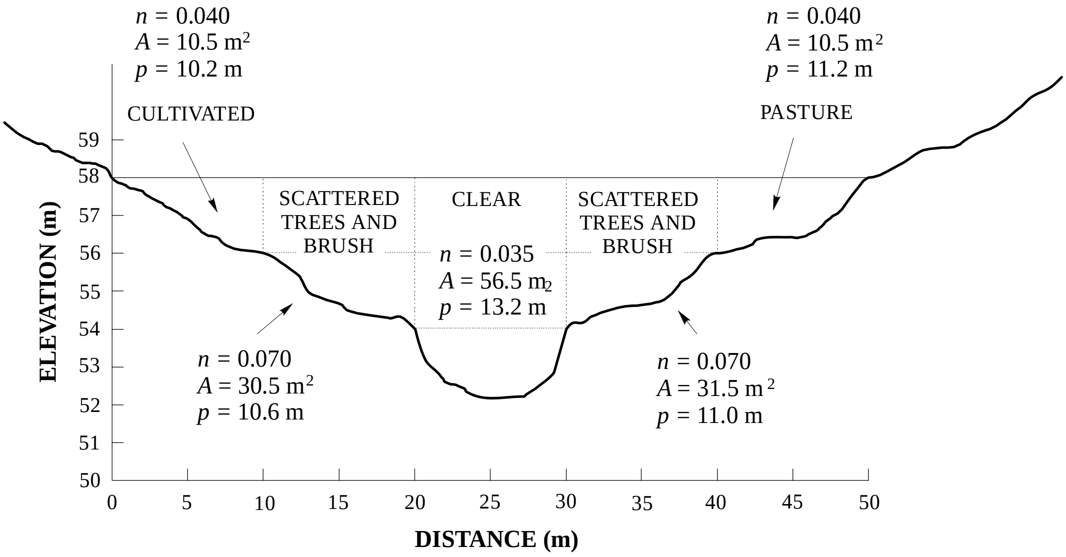

Figure 6 – Stream Cross Section at Gauging Site
Stage Height = 58.0 m

STAGE DISCHARGE COMPUTATIONS						
Slope S (m/m) 0.00050						
Subsection Chainage (m)	**Manning's Coefficient**	**Area (m^2)**	**Wetted Perimeter (m)**	**Hydraulic Radius (m)**		**Discharge (m^3/s)**
	n	A	p	$R = A/p$	$R^{2/3}$	Q
0 to 10	0.040	10.5	10.2	1.0294	1.0195	5.98
10 to 20	0.070	30.5	10.6	2.8774	2.0230	19.71
20 to 30	0.035	56.5	13.2	4.2803	2.6362	95.16
30 to 40	0.070	31.5	11.0	2.8636	2.0166	20.29
40 to 50	0.040	10.5	11.2	0.9375	0.9579	5.62
					TOTAL	146.77

Table 3 – Stage Discharge Computations
Stage Height = 58.0 m

The total discharge can then be found by adding the discharges for each subsection. This process can be repeated for other stage heights and a *Stage-Discharge Rating Curve* drawn.

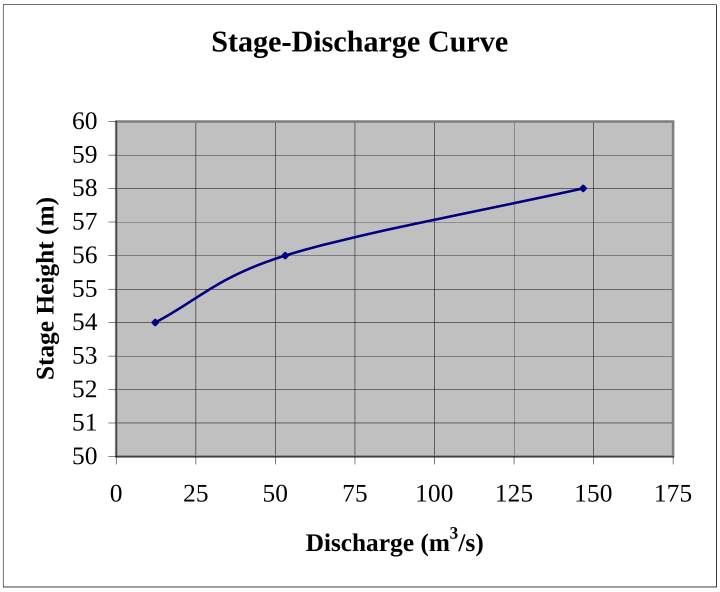

Figure 7 – Stage-Discharge Curve

Care should be exercised in the survey of the cross section and determination of the stream gradient if realistic results are to be obtained with this method. A matter of prime importance in using the Slope-Area Method is the selection of appropriate retardance factors (Manning roughness coefficients *n*) of the main channel and the floodplains. Both may be subject to extreme variations with vegetation growth and depth of flow.

Brief historical outlines of some famous water engineers mentioned in these notes.

Chézy, Antoine de (b. 1718, Châlons-sur-Marne, France – d. Oct. 5, 1798, Paris), French hydraulic engineer and author of a basic formula for calculating the velocity of a fluid stream. One of the group of brilliant engineers produced by the French School of Bridges and Highways (École des Ponts et Chaussées) in the 18th century, Chézy carried out studies in connection with the construction of French canals, notably in 1764 the difficult project of the Canal de Bourgogne, uniting the Seine and Rhône basins. Chézy was exceptionally modest and even timid, and, though he served as right-hand man to the celebrated bridge-builder Jean-Rodolphe Perronnet, whose Pont de la Concorde in Paris he completed (1795), his genius was only tardily recognized; he was appointed director of the School of Bridges and Highways in the last year of his life.

Darcy, Henri-Philibert-Gaspard (b. June 10, 1803, Dijon, France – d. Jan. 3, 1858, Paris), French hydraulic engineer who first derived the equation (now known as Darcy's Law) that governs the laminar (non-turbulent) flow of fluids in homogeneous, porous media and who thereby established the theoretical foundation of groundwater hydrology. After studying in Paris, Darcy returned to his native city of Dijon, where he was entrusted with the design and construction of the municipal water supply system. During the course of this work, he conducted experiments on pipe flow and demonstrated that resistance to flow depended on the surface roughness of the pipe material, which previously had not been considered a factor. Planning to use the technique of water purification by filtration through sand, he also studied cases in which the pipe was filled with sand. From the data gathered, he derived the law that bears his name. The darcy is the standard unit of permeability.

Bazin, Henri-Émile (b. Jan. 10, 1829, Nancy, France – d. Feb. 7, 1917, Dijon), engineer and member of the French Corps des Ponts et Chaussées ("Corps of Bridges and Highways") whose contributions to hydraulics and fluid mechanics include the classic study of water flow in open channels. Bazin worked as an assistant to the noted hydraulic engineer H.-P.-G. Darcy (1803-58), whose program of tests on resistance to water flow in channels Bazin finished after Darcy died. The results were published in 1865. Bazin then carried his study over into the problem of wave propagation and the contraction of fluid flowing through an orifice. In 1854 he enlarged the Canal de Bourgogne and made it profitable for commercial navigation. In 1867 he suggested the use of pumps for dredging rivers, leading to the construction of the first suction dredgers. Bazin became chief engineer of the Corps des Ponts et Chaussées in 1875 and was placed in charge of the Bourgogne canal system; he became inspector general in 1886. He retired in 1900 and was elected to the French Academy of Sciences in 1913.

Ganguillet, E.O. & Rudolph **Kutter.** A complicated empirical formula for the discharge of streams resulted from the studies of Andrew Atkinson Humphreys and Henry Larcom Abbot in the course of the Mississippi Delta Survey of 1851-60. Their formula contained no term for roughness of channel and on this and other grounds was later found to be inapplicable to the rapidly flowing streams of mountainous regions. In 1869, Emile-Oscar **Ganguillet** and Rudolph **Kutter** developed a more generally applicable discharge equation following their studies of flow in Swiss mountain streams. Toward the end of the century, systematic studies of the discharge of streams had become common. In the United States the Geological Survey, following its establishment in 1879, became the principal agency for collecting and publishing data on discharge, and by 1906 stream gauging had become nationwide.

Manning, Robert (1816-1897). Robert Manning was born in Normandy but shifted to Waterford, Ireland in 1826 with his mother after the death of his father. Manning worked as an accountant from 1834 to 1845 when he was drafted into the Arterial Drainage Division of the Irish Office of Public Works; serving as a clerk, accountant and draftsman until appointed as an assistant engineer in 1846. And finally as District Engineer from 1848 to 1855. Manning left the Public Works and entered the employ of the Marquis of Downshire in 1855, returning to the Office of Public Works (as assistant Chief Engineer) in 1869 after the death of the Marquis. He was appointed Chief Engineer in 1874 and held that position until his retirement in 1891. Manning received no formal training in engineering and fluid mechanics but studied the works of the French pioneers in hydraulics: Chezy, Du Bat and Eytelwein as well as his contempories: Darcy, Bazin and Kutter. His background in accounting motivated his drive to reduce problems to their simplest form and he expressed disdain for complex mathematical formulations. Manning developed his equation by examining the then current formulae (and observational data) for velocity of open channel flow, arriving at the expression $V = C\,R^x S^{1/2}$ (*V* = velocity, *C* a coefficient, *S* = slope and *R* = wetted surface area or hydraulic radius). Manning determined that the value of the exponent *x* = 0.666 and in 1855 arrived at the equation $V = C\,R^{2/3} S^{1/2}$. Manning thought this equation too difficult to use, since it required the evaluation of a two-thirds power, as well as being dimensionally incorrect. Other practitioners found his equation most useful. Correspondence with others lead Manning to note that his coefficient *C* corresponded closely with the inverse of *n* (a coefficient of roughness) used in the more complex formula by Ganguillet & Kutter; both formula giving practically identical results for observed flow conditions. The simplicity of Manning's formula (presented in his paper "On the Flow of Water in Open Channels and Pipes" published in *Transactions of the Institution of Civil Engineers of Ireland*, 1891) has lead to its wide adoption. It is often given in the form $V = \frac{k}{n} R^{2/3} S^{1/2}$ where *k* = 1.486 for English units and *k* = 1 for SI units.

[Information on Manning obtained from:
Fischenich, C., April 2000, *Robert Manning (A Historical Perspective)*
http://www.wes.army.mil/el/emrrp/pdf/sr10.pdf]

WEIR METHOD

Weirs are artificial barriers built across streams, dams can also be considered as (broad crested) weirs, and they have long been used to measure the flow of water in streams and channels. Figure 9 shows a sectional view of water in a channel flowing over a weir. For stream gauging, the discharge is determined by measuring the height H of the *still water* surface above the *sill* or the *crest* of the weir. This measurement is made at a distance upstream of the weir, where the *drawdown* will have negligible effect. The amount of drawdown at the crest is approximately $0.15H$. The weir should be vertical and the crest should be horizontal with a sharp upstream edge. The thickness of the crest should be such that the *nappe* springs clear of the weir; weirs with this characteristic are known as *sharp crested* weirs and are the only ones considered in these noters. [Weirs where the nappe does not spring clear are known as *broad crested* weirs and coefficients developed for sharp crested weirs do not necessarily apply to broad crested weirs.]

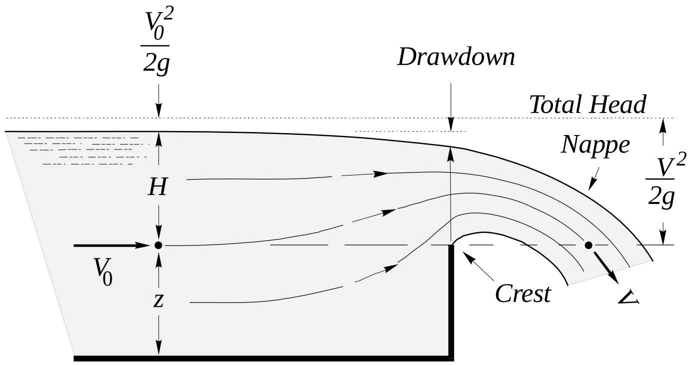

Figure 9 – Water flowing over a weir

z is the height of the weir above the channel or stream bed. At some distance upstream of the weir (where the drawdown is zero) the stream depth is $H + z$, the mean velocity is V_0 and the velocity head is $\frac{V_0^2}{2g}$. The velocity V of the nappe is greater than V_0 and the velocity head has increased.

A weir constructed across a stream or channel for gauging purposes may also be in the form of a *notch*. Figure 10 shows a schematic view of a *rectangular notched weir* with the nappe springing clear of the crest, and of the sides of the notch. Such weirs are known as *contracted weirs* and in the case of a rectangular notch, there are two contractions (the two sides of the notch).

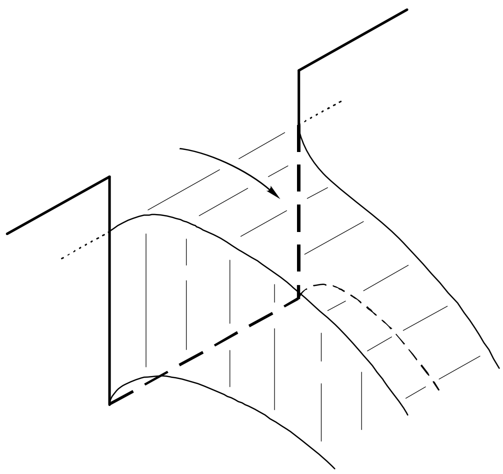

Figure 10 – Water flowing over rectangular notched weir

There are several types of Weirs or Notches in practical use; Figure 11 (i) rectangular, (ii) V-shaped, (iii) trapezoidal (known as a Cipoletti weir) and (iv) stepped. In each case, H is the height of the water over the crest of the weir (or the apex of the V-shaped notch) and B is the breadth of the weir.

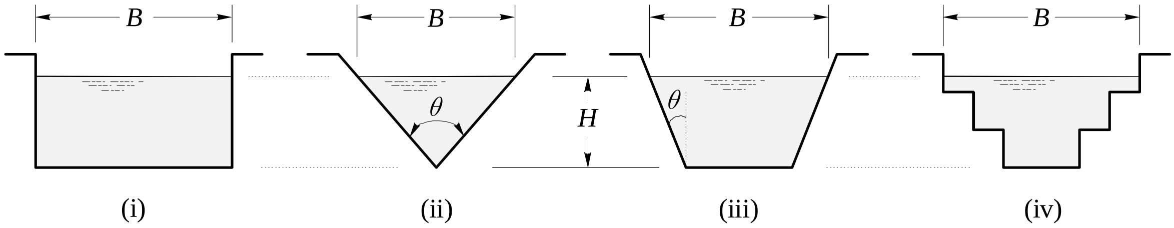

Figure 11 – Types of Weirs or Notches

Basic Rectangular Weir Formulas

To derive a discharge equation for a rectangular weir, consider Figure 12, which shows a weir of height z across a rectangular channel. The breadth of the weir B is equal to the width of the channel. Weirs of this type are called *suppressed* ie, there are no side contractions (a rectangular notched weir has two side contractions). An element of area in the plane of the crest is $dA = B\,dh$. Assuming the velocity of water flowing through the area is $v = \sqrt{2gh}$, the discharge through this elemental area is $dQ = v\,dA = \sqrt{2gh}\,B\,dh$. This differential relationship can be rearranged in the form

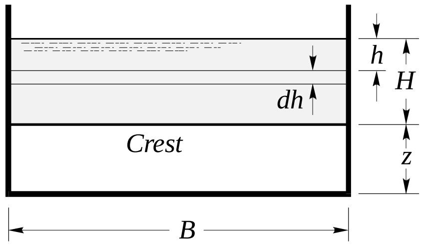

Figure 12 – Rectangular, sharp crested weir without end contractions

$$dQ = B\sqrt{2g}\,h^{1/2} \qquad (6)$$

The theoretical total discharge over the weir can be obtained by integrating (6) over the whole area between $h = 0$ and $h = H$ giving an *ideal* discharge

$$Q_{ideal} = B\sqrt{2g}\int_0^H h^{1/2}\,dh = \frac{2}{3}\sqrt{2g}\,B\,H^{3/2} \qquad (7)$$

However, this ideal discharge will be decreased slightly by fluid friction and much more by other factors. Considering Figures 9 and 10 it is clear that the area of the stream in the plane of the crest is less than BH, due to the drawdown at the surface and crest contraction below. To correct for these factors an experimentally determined coefficient of discharge C_d is introduced, giving the basic weir formula (Daugherty & Ingersoll 1954)

Rectangular weir formula $$Q = C_d\,\frac{2}{3}\sqrt{2g}\,B\,H^{3/2} \qquad (8)$$

If a weir of this type is to be used to gauge stream discharge, then it must be calibrated experimentally. From (8) the discharge for any given weir is $Q = k\,H^{3/2}$ where

$$k = \frac{2}{3}C_d\sqrt{2g}\,B$$

Then by measuring the discharge Q for various heads H the value of k may be obtained by plotting Q (on the *y-axis*) against $H^{3/2}$ (on the *x-axis*). A line of best fit through the data and passing through the origin will allow k to be determined.

In 1848 – 1852 at Lowell, Massachusetts, USA, James B Francis made precise investigations of the flow of water over large weirs[1]. His observations showed the flow of water varied as $H^{1.47}$ but he adopted $H^{3/2}$ for greater convenience. He then selected a constant average value of $C_d = 0.622$, so that for an average value of $g = 9.81\text{ m/s}^2$ the rectangular weir formula (8) is given as

Francis' weir formula $$Q = 1.84\,BH^{3/2} \tag{9}$$

When the breadth *B* of the weir is less than that of the channel, ie, the weir is a rectangular notch, Figure 11, type (i), Francis concluded from his experiments that the effect of each side contraction (and a rectangular notch has two) is to reduce the width of the nappe by 0.1*H*. If *n* = the number of contractions, which may be 2, 1, or 0, Francis' formula becomes (Daugherty & Ingersoll 1954)

Francis' contracted weir formula $$Q = 1.84\left(B - 0.1nH \right) H^{3/2} \tag{10}$$

> **Francis**, James Bicheno (b. May 18, 1815, Southleigh, Devon, Eng. – d. Sept. 18, 1892, Lowell, Mass., U.S.), British-American hydraulic engineer and inventor of the mixed-flow, or Francis turbine (a combination of the radial- and axial-flow turbines) that was used for low-pressure installations. In 1833 Francis went to the United States and was hired by the engineer G.W. Whistler to help construct the Stonington (Conn.) Railway. In Lowell he joined the Proprietors of the Locks and Canals on the Merrimack River as a draftsman and at age 22 became chief engineer of the company. In his 40 years of managing the company's waterpower interests and acting as a consulting waterpower engineer to factories, he contributed greatly to the rise of Lowell as an industrial centre. He also investigated timber preservation, the testing and design of cast-iron girders, and fire protection systems. In addition to the Francis turbine, he is known for his formulas for the flow of water over weirs and many other hydraulic studies. Francis wrote more than 200 technical papers and, although unschooled, was considered one of the foremost civil engineers of his time.
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V-notch Weir Formula

A V-notch weir is also used for stream gauging and has a slight advantage over a rectangular weir (or rectangular notch). It is known that the coefficient of discharge C_d, which must be determined experimentally, varies with the head *H* of the water over the crest. This variation is more pronounced with a rectangular notch than with a V-notch, thus making the V-notched weir easier to calibrate and more capable of determining an accurate discharge over a wider range of heads (Fox 1974).

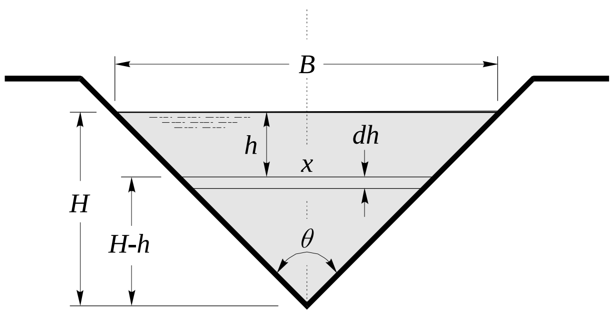

Figure 13 – V-Notch of Weir

In a similar manner to the rectangular weir, the discharge through the elemental area *dA* is

[1] Francis, J. B., 1909. *Lowell Hydraulic Experiments,* 5th ed, Van Nostrand Co, New York.

$$dQ = x\,dh\sqrt{2gh}$$

From similar triangles $\dfrac{x}{H-h} = \dfrac{B}{H}$ but $\dfrac{B}{H} = 2\tan\dfrac{\theta}{2}$ giving $x = 2(H-h)\tan\dfrac{\theta}{2}$.

Thus $$dQ = 2(H-h)\tan\frac{\theta}{2}\sqrt{2g}\,h^{1/2}\,dh \tag{11}$$

Integrating (11) over the whole area between $h = 0$ and $h = H$ gives an *ideal* discharge

$$\begin{aligned}
Q_{ideal} &= 2\sqrt{2g}\,\tan\frac{\theta}{2}\int_0^H (H-h)\,h^{1/2}\,dh \\
&= 2\sqrt{2g}\,\tan\frac{\theta}{2}\left[H\int_0^H h^{1/2}\,dh - \int_0^H h^{3/2}\,dh\right] \\
&= 2\sqrt{2g}\,\tan\frac{\theta}{2}\left(\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right) \\
&= \frac{8}{15}\sqrt{2g}\,\tan\frac{\theta}{2}H^{5/2}
\end{aligned}$$

The ideal discharge will be decreased slightly due to fluid friction and other factors, and introducing a coefficient of discharge C_d gives the basic V-notch weir formula (Daugherty & Ingersoll 1954)

V-notch weir formula $$Q = \frac{8}{15}C_d\sqrt{2g}\,\tan\frac{\theta}{2}H^{5/2} \tag{12}$$

As for rectangular weirs, the coefficient of discharge may be determined by experiment.

For a right-angled weir $\theta = 90^\circ$ and for heads above 0.06 m Addison (1941) states that C_d has a nearly constant value of 0.593. Inserting this value into (12) with $g = 9.81\ \text{m/s}^2$ gives

Right-angled V-notched weir $$Q = 1.40\,H^{5/2} \tag{13}$$

VOLUMETRIC METHOD

The most accurate method for measuring small stream discharges Q is the volumetric method. This is simply done by measuring the time taken to fill a container to a known volume. The equipment required is a calibrated container and a stop watch.

Calibration of the container can be achieved by weighing the container with varying amounts of water in it, noting its level. A container can also be calibrated by adding known volumes of water by increments.

Volumetric measurements should be made where the flow is concentrated into a narrow stream or an artificial control, where the flow is confined to a narrow width of a shaped weir crest. Sometimes it is necessary to place a trough or funnel against the artificial control to carry the water to the calibrated container. The measurement should be carried out three or four times, to be certain that errors have not been made and that results are consistent. It is good practice to then mean the accepted readings.

Volumetric measurements of discharge may be used to calibrate notched weirs.

Continuous Measurement of Stream Discharge

Although individual and separate discharge measurements may be necessary for certain engineering projects, the flow characteristics of a catchment or stream cannot be fully understood unless continuous records of flow rate and volume of flow are taken over a number of years. In Victoria, this work is or has been the responsibility of government and semi-government authorities and some historical background is useful.

The **State Rivers and Water Supply Commission** (SR&WSC) of Victoria was formed to bring order to water supply in Victoria. The drought of the 1870's had highlighted the need for adequate water resources. Farmers lobbied for Government action, resulting in the Irrigation Act of 1886 and the establishment of irrigation and water supply trusts. By the turn of the century, the trusts had failed; due in part to a lack of centralised administrative control, and rising debt. The Water Act of 1905 abolished the irrigation trusts, and provided for the establishment of the SR&WSC on 1 May, 1906. Its brief was to develop, build, control and maintain surface water resources. River gauging began in 1865, on the Murray River at Mildura, and by 1900, some 30 streams had records spanning a decade or more. In 1984 the SR&WSC was abolished and in its place, the **Rural Water Commission** of Victoria was established. By this time, there were more than 500 stream gauges (gauging sites) in operation. In 1990, the Rural Water Commission was moved from the Department of Water Resources to become part of the Department of Conservation and Environment. In 1992, its operations were taken over by the **Rural Water Corporation** (RWC), a corporate body established outside the Victorian Public Service and in 1995 the RWC was disbanded, with its five regions becoming autonomous water authorities. By this stage there were approximately 700 stream gauging sites across Victoria; equivalent to a density of one station per 270 km2 of area within the state producing measurable surface water runoff.

Continuous records can be taken by either the Area Velocity or Weir methods. If weirs are used, then daily (or with rapid variation, more frequently) readings are required of the head over weir, taken either by onsite inspection or by automatic recording methods. If the Area Velocity method is adopted, current meter readings should be taken on various occasions during times of low, average and flood flow. The current meter readings are correlated with gauge height readings to relate cross section areas with velocities of flows and discharges calculated. From observed gauge heights and calculated discharges, a *Stage-Discharge Rating Curve* can be obtained by plotting gauge height (on the *y-axis*) versus discharge (on the *x-axis*). For any other gauge height readings, whether taken by an observer or by an automatic gauge recorder, the corresponding discharge value may be determined. An example of a Stage-Discharge Rating Curve is shown in Figure 7.

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