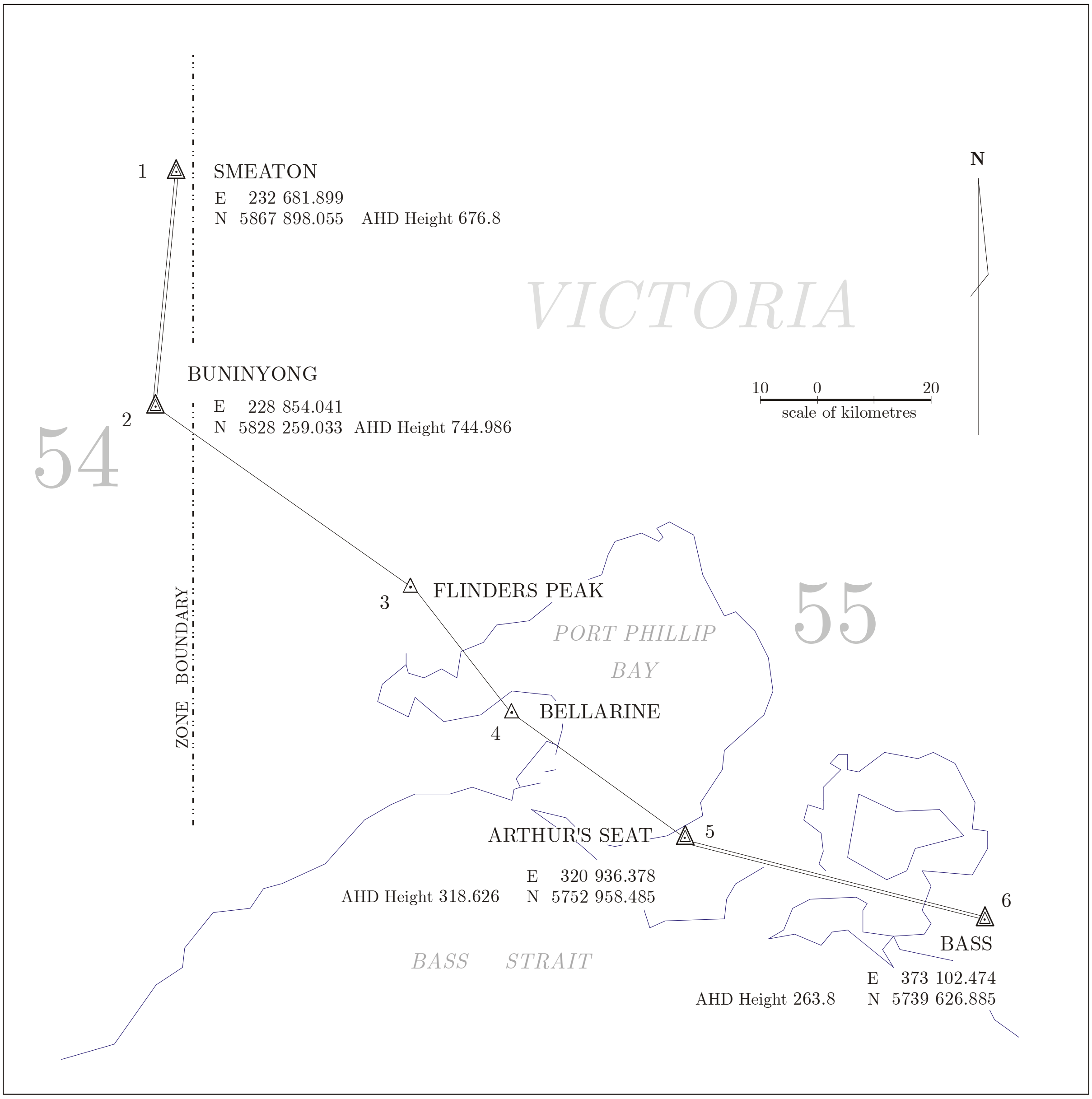
Traverse Computation on the Ellipsoid and on the Universal Transverse Mercator Projection



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First printing March 2005
This printing with minor amendments March 2010

TRAVERSE COMPUTATION ON THE ELLIPSOID AND ON THE UNIVERSAL TRANSVERSE MERCATOR PROJECTION

These notes describe in detail the methods and formulae used in reducing traverses of measured directions and distances on the Earth's surface to: (i) a set of quasi-measurements on the reference ellipsoid and then (ii) a set of derived (plane) directions and (plane) distances on the Universal Transverse Mercator projection.

As an example of the reductions required a traverse between trigonometric stations *Buninyong, Flinders Peak, Bellarine* and *Arthur's Seat* in Victoria is used. This traverse should be well known to surveyors in Australia as it has been used to demonstrate reduction techniques in The Australian Map Grid Technical Manual (NMC 1972), The Australian Geodetic Datum Technical Manual (NMC 1985) and the Geocentric Datum of Australia Technical Manual (ICSM 2002). In the latter publication the geodetic and grid coordinates are Geocentric Datum of Australia 1994 (GDA94) and Map Grid Australia (MGA94) values respectively. In the earlier publications the coordinates were Australian Geodetic Datum 1966 (AGD66) and Australian Map Grid (AMG66) values. Figure 1 shows a diagram of the *Buninyong-Arthur's Seat* traverse with the original AMG66 grid coordinates of the fixed stations in the traverse. The coordinates in brackets are MGA94 and have been derived from the AMG66 coordinates using the transformation program *GDAit* Version 2.2 (18/10/01) and data file A66 National (13.09.01).gsb; these transformed values will be used in these notes.

In The Australian Map Grid Technical Manual (NMC 1972) the original measurements were a set of quasi-observations on the ellipsoid, *spheroidal distances* and (clockwise) *spheroidal angles*, shown below in Table 1. In these notes we call these quasi-observations *geodesic distances* and *geodesic angles*.

	Station / Spheroidal Distance	Spheroidal angle	
1	Smeaton		
2	Buninyong (2-3) 54972.161 m	1-2-3	119°47´10.06˝
3	Flinders Peak (3-4) 27659.183 m	2-3-4	196°43´49.44˝
4	Berllarine (4-5) 37175.169 m	3-4-5	163°45´32.33˝
5	Arthur's Seat	4-5-6	158°34´37.46˝
6	Bass		

Table 1. Spheroidal distances and angles of the *Buninyong-Arthur's Seat* traverse

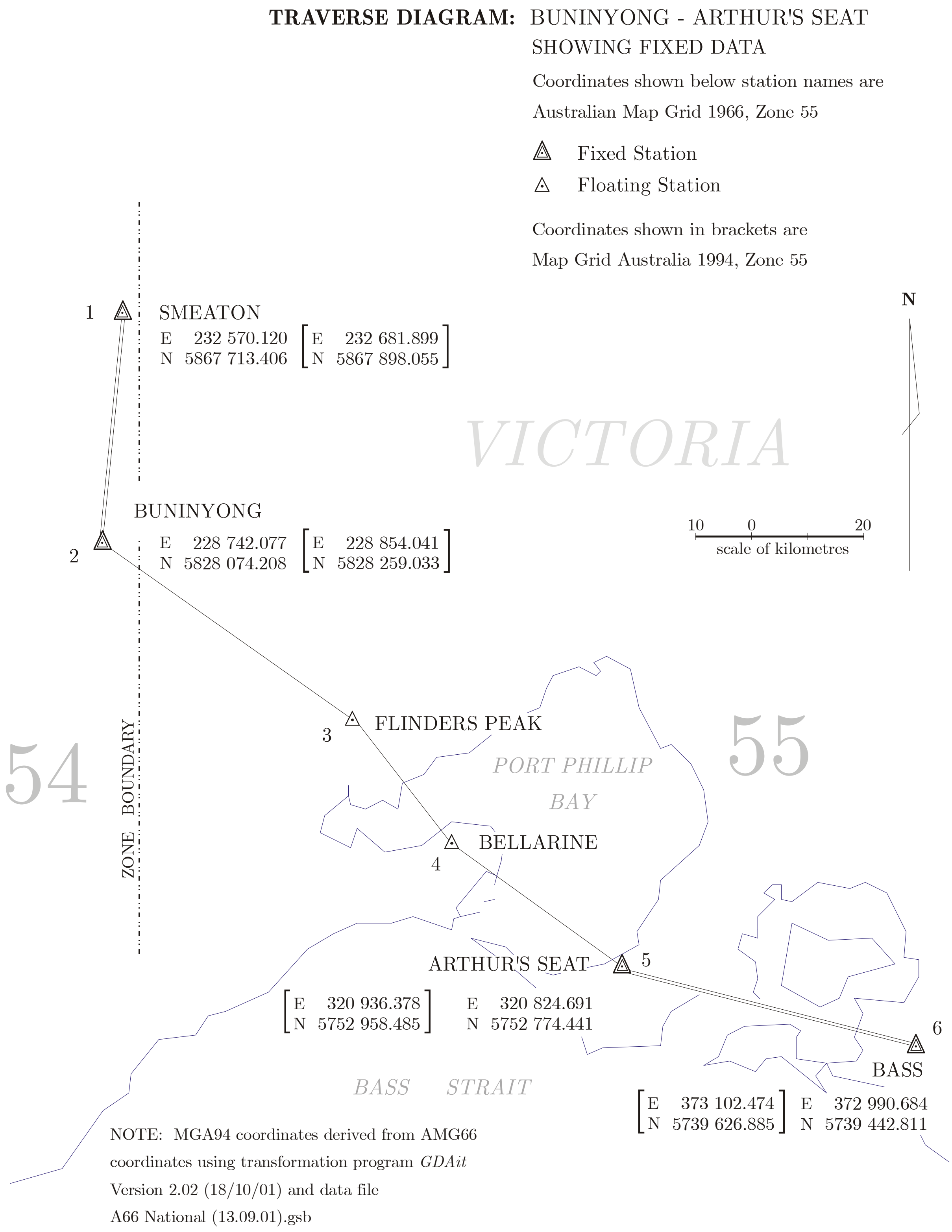

Figure 1. *Buninyong-Arthur's Seat* traverse

Data from: *The Australian Map Grid Technical Manual,* Special Publication 7, National Mapping Council of Australia, 1972.

REDUCTION OF TRAVERSE MEASUREMENTS TO THE ELLIPSOID

The traverse measurements to be considered are:

1. Horizontal directions measured with a theodolite,
2. Vertical circle observations measured with a theodolite,
3. Slope distances (or chord distances) derived from measurements made with Electronic Distance Measuring (EDM) equipment.

Traverse measurements are made between points on the Earth's terrestrial surface and traverse stations $P_1, P_2, \dots, P_k$ have complimentary points $Q_1, Q_2, \dots, Q_k$ on the surface of the reference ellipsoid; Q is the projection of P via the ellipsoid normal. The reduction of measurements to the ellipsoid means applying a series of corrections to the measurements made between points P_k to obtain a set of quasi-measurements between points Q_k on the ellipsoid. The corrections may be divided into *gravimetric* and *geometric* corrections; gravimetric corrections applied first, followed by geometric corrections.

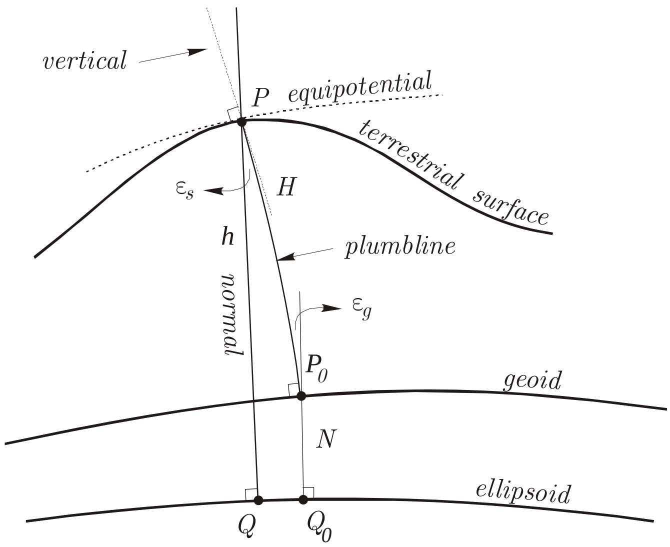

Figure 2. Projection of P on the Earth's surface to Q on the ellipsoid and P_0 on the geoid. Note that P, Q, P_0 and Q_0 do not lie in the same vertical plane since the plumbline is a 3D space curve.

Gravimetric corrections are applied to theodolite observations only. The rotational axis of a theodolite (that is correctly levelled and in adjustment) is coincident with the *vertical* at P and the horizontal plane of the theodolite is tangential to the equipotential surface of the Earth's gravity field passing through P. In general, the vertical and the ellipsoid normal do

not coincide and gravimetric corrections are applied to the observations to create a set of measurements with respect to the ellipsoid normal and geodetic horizon plane at P. The geodetic horizon plane at P is a plane parallel to the plane tangential to the ellipsoid at Q. Gravimetric corrections involve the deflection components ξ (meridian plane) and η (prime vertical plane) which must be obtained from a geoid model such as AUSGeoid98.
Gravimetric corrections are generally very small and are often ignored. In these notes they will be computed and applied and the reader may gauge their applicability to other traverse reductions.
Geometric corrections are applied to both theodolite and distance measurements.

SYMBOLS

THE GREEK ALPHABET

Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Υ	υ
Epsilon	E	ε	Nu	N	ν	Phi	Φ	ϕ φ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ ϑ	Pi	Π	π	Omega	Ω	ω

α = azimuth, clockwise from true north 0° to 360°
α_A, α_G = astronomic azimuth and geodetic azimuth
α_{12}, α_{21} = azimuth from point 1 to point 2 and azimuth from point 2 to point 1
β = grid bearing measured clockwise from Grid North, $\beta = \alpha + \gamma$
γ = angle of refraction
γ = grid convergence, positive East of the central meridian and negative West
δ = arc-to-chord correction with sign defined by $\theta = \beta + \delta$
ε = deflection of the vertical in direction α and $\varepsilon = \xi\cos\alpha + \eta\sin\alpha$
$\varepsilon_s, \varepsilon_g$ = deflections of the vertical at the Earth's surface and the geoid respectively
η = deflection of the vertical in the prime vertical plane
η_s, η_g = deflection of the vertical in the prime vertical plane at the earth's surface and the geoid
θ = plane bearing measured clockwise from Grid North, $\theta = \beta + \delta$
ν = radius of curvature in prime vertical plane
ν_1, ν_2 = radius of curvature of ellipsoid in prime vertical plane at points 1 and 2

ν_m	=	$(\nu_1 + \nu_2)/2$
ξ	=	deflection of the vertical in the meridian plane
ξ_s, ξ_g	=	deflections of the vertical in the meridian plane at the Earth's surface and the geoid
ρ	=	radius of curvature of ellipsoid in meridian plane
ρ_1, ρ_2	=	radius of curvature of ellipsoid in meridian plane at points 1 and 2
ρ_m	=	$(\rho_1 + \rho_2)/2$
ϕ	=	geodetic latitude, negative south of the equator
ϕ_1, ϕ_2	=	latitude at points 1 and 2 respectively
ϕ_m	=	$(\phi_1 + \phi_2)/2$
s	=	geodesic distance (also ellipsoidal or spheroidal distance)
$a,\ b$	=	major and minor semi-axes of the ellipsoid
e^2	=	square of the (first) eccentricity of the ellipsoid = $(a^2 - b^2)/a^2$
e'^2	=	square of the second eccentricity of the ellipsoid = $(a^2 - b^2)/b^2$
f	=	flattening of ellipsoid = $(a - b)/a$
h	=	ellipsoid height (height above ellipsoid measured along normal)
h_1, h_2	=	ellipsoidal heights at points 1 and 2 respectively
H	=	orthometric height (height above geoid measured along plumbline)
K	=	Line Scale Factor
k	=	coefficient of refraction
k	=	Point Scale Factor
k_0	=	central meridian scale factor = 0.9996
L	=	plane distance
N	=	$h - H$ = geoid-ellipsoid separation
R	=	$\sqrt{\rho\nu}$ = mean radius of curvature
R_α	=	radius of curvature of a normal section in a given azimuth
r_m^2	=	$\rho\nu k_0^2$ for $\phi_m = (\phi_1 + \phi_2)/2$
z	=	zenith distance (vertical angle measured from the zenith)

REDUCTION FORMULA

The derivations of the following equations can be found in selected texts and references. No attempt is made here to explain the derivations and the reader is directed to the references for derivations and detailed discussion.

GRAVIMETRIC CORRECTIONS TO THEODOLITE OBSERVATIONS

$$\text{corrected direction} = \text{observed direction} + \left\{ \frac{-\xi_s \sin\alpha_A + \eta_s \cos\alpha_A}{\tan z_A} \right\} \tag{1}$$

$$\text{corrected zenith distance} = \text{observed zenith distance} + \left\{ \xi_s \cos\alpha_A + \eta_s \sin\alpha_A \right\} \tag{2}$$

In equations (1) and (2) observed direction and zenith distance are measurements made at P with a theodolite (that is correctly levelled and in adjustment) whose rotational axis is coincident with the vertical at P and the horizontal plane of the theodolite is tangential to the equipotential surface of the Earth's gravity field passing through P. Corrected direction and zenith distance are quasi-measurements made at P with a theodolite whose rotational axis is coincident with the normal at P and the horizontal plane of the theodolite is the geodetic horizon plane at P (the geodetic horizon plane at P is a plane parallel to the plane tangential to the ellipsoid at Q).

ξ_s and η_s are deflections of the vertical (meridian and prime vertical respectively) and the subscript s indicates that these are deflections at the terrestrial surface at P, see Figure 2 where $\varepsilon = \xi \cos\alpha + \eta \sin\alpha$. Deflections can be determined by observations, but are usually computed from geoid models such as AUSGeoid98 in which case they are deflections of the vertical at the geoid, denoted by ξ_g and η_g. The differences between ξ_s, η_s and ξ_g, η_g are due to the gravimetric effects of the terrain between the geoid and the terrestrial surface in the vicinity of P. They are difficult to model but they are usually very small (Featherstone & Rüeger 2000). In these notes the differences are ignored and $\xi = \xi_s = \xi_g$, $\eta = \eta_s = \eta_g$ is assumed.

α_A is azimuth and the subscript A indicates that it is astronomic azimuth, i.e., an angle with respect to the observer's astronomic meridian (a meridian plane containing the north and south poles and the vertical at P). α_G is azimuth and the subscript G indicates that it is geodetic azimuth, i.e., an angle with respect to the observer's geodetic meridian (a meridian plane containing the north and south poles and the ellipsoid normal at P). The difference between α_G and α_A is small and for the purposes of reduction equations (1) and (2) it can be assumed that $\alpha = \alpha_A = \alpha_G$.

REFERENCES

Featherstone, W.E. and Rüeger, J.M., 2000. 'The importance of using deflections of the vertical for reduction of survey data to a geocentric datum', *The Trans Tasman Surveyor,* Vol. 1, No. 3, pp. 46-61, December 2000. See also "Erratum" in *The Australian Surveyor,* Vol. 47, No. 1, p.7, June 2002.

Heiskanen, W.A. and Moritz, H., 1967. *Physical Geodesy,* W.H. Freeman & Co., London, pp. 184-88.

Vanicek, P. and Krakiwsky, E., 1986. *Geodesy: The Concepts,* North-Holland, Amsterdam.

RMIT Lecture Notes, *Relationship Between Astronomic Coordinates ϕ_A, λ_A, H and Geodetic Coordinates ϕ_G, λ_G, h,* 8 pages.

GEOMETRIC CORRECTIONS TO THEODOLITE OBSERVATIONS

SKEW–NORMAL CORRECTION TO GEODETIC DIRECTIONS ON AN ELLIPSOID

$$\text{correct normal section dir'n} = \text{observed normal section dir'n} + \left\{ \frac{h_2}{2\rho_m} e^2 \sin 2\alpha_{12} \cos^2 \phi_2 \right\} \quad (3)$$

where

h_2 is the ellipsoidal height of station 2

$e^2 = f(2-f)$ is the (first) eccentricity squared and f is the ellipsoid flattening

$\rho = \dfrac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}}$ is the radius of curvature of the ellipsoid in the meridian plane

$\rho_m = \dfrac{\rho_1 + \rho_2}{2}$

α_{12} is the azimuth between points 1 and 2

ϕ_2 is the latitude of point 2

In equation (3) the skew-normal correction, the term in braces { }, will be in radians. The conversion from radians to seconds of arc is: seconds of arc = radians $\times \dfrac{180}{\pi} \times 3600$.

The correction given by equation (3) is accurate to at least 0.001" for lines up to 100 km in length for targets 1000 m above the ellipsoid.

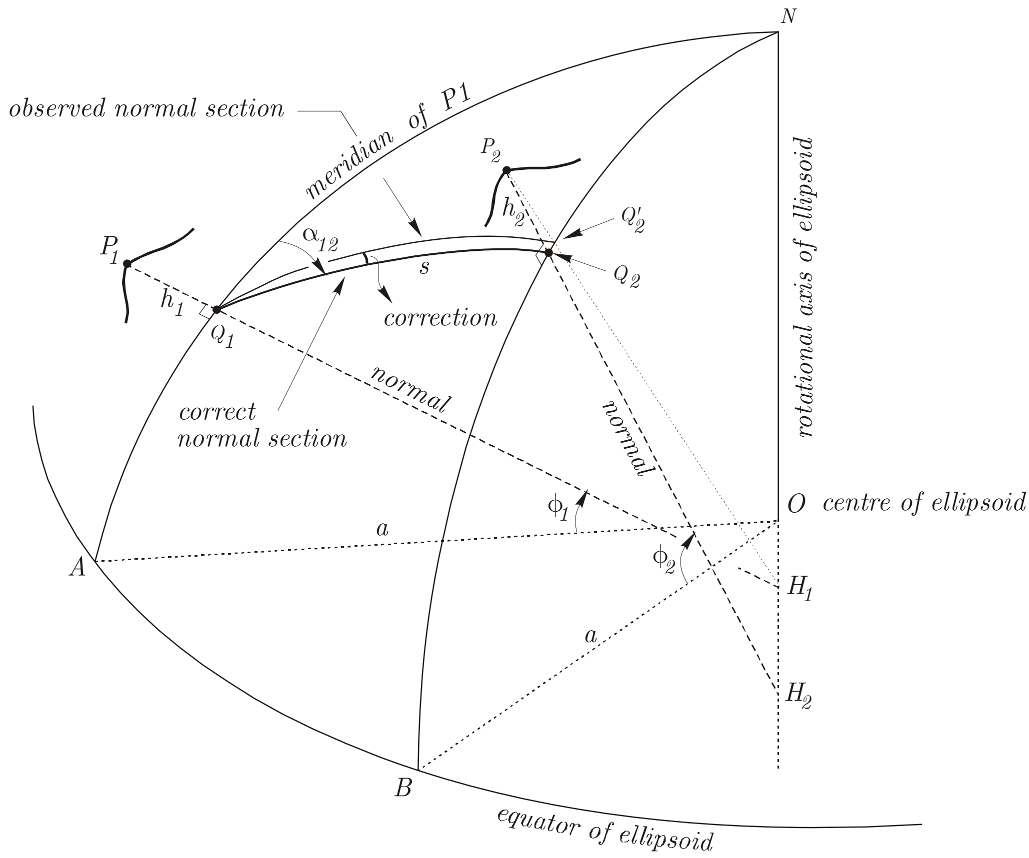

Figure 3. Skew-normal correction

A plane containing the normal at P_1 (height h_1 above the ellipsoid) and P_2 at height h_2 intersects the ellipsoid along a normal section curve Q_1Q_2'. This is the observed normal section direction. But the projection of P_2 onto the ellipsoid (via the normal at P_2) is the point Q_2 and the plane containing the normal at P_1 and the point Q_2 intersects the ellipsoid along the normal section Q_1Q_2. This is the correct normal section direction. The skew-normal correction is also called the height of target correction.

CORRECTION FROM THE NORMAL SECTION DIRECTION TO THE GEODESIC DIRECTION

$$\text{geodesic direction} = \text{normal section direction} + \left\{ -\frac{s^2}{12\nu_m^2} e^2 \sin 2\alpha_{12} \cos^2 \phi_m \right\} \qquad (4)$$

where

s is the geodesic distance on the ellipsoidal between points 1 and 2

$e^2 = f(2-f)$ is the (first) eccentricity squared and f is the ellipsoid flattening

$\nu = \dfrac{a}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}}$ is the radius of curvature of the ellipsoid in the prime vertical plane

$\nu_m = \dfrac{\nu_1 + \nu_2}{2}$

α_{12} is the azimuth between points 1 and 2

$\phi_m = \dfrac{\phi_1 + \phi_2}{2}$ is the mean of latitudes of points 1 and 2

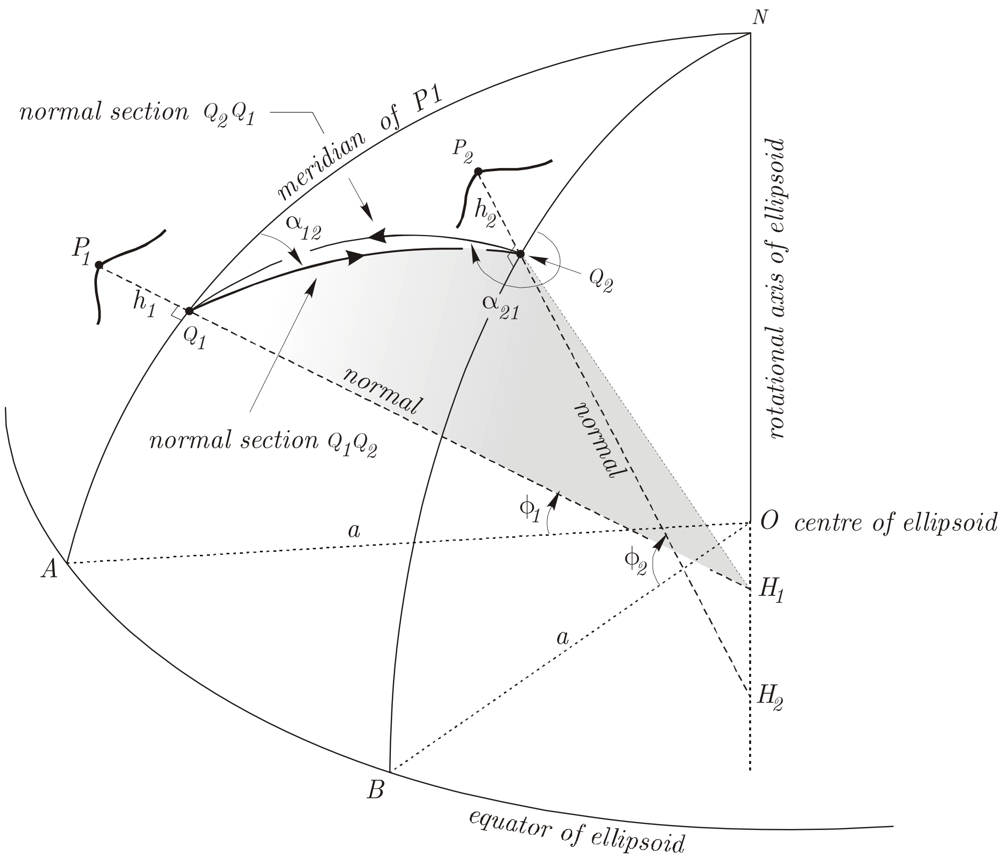

Figure 4. Reciprocal normal sections on the ellipsoid

A plane containing the ellipsoid normal at P_1 and the point Q_2 (the projection of P_2 on the ellipsoid) intersects the surface of the ellipsoid along the normal section curve Q_1Q_2. The reciprocal normal section curve Q_2Q_1 (the intersection of the plane containing the normal at P_2 and the point Q_1 with the ellipsoidal surface) does not in general coincide with the normal section curve Q_1Q_2 although the distances along the two curves are for all practical purposes the same. Hence there is not a unique normal section curve between Q_1 and Q_2.

A *geodesic* is a unique curve on the surface of an ellipsoid and is the line of shortest distance between Q_1 and Q_2. In general, a geodesic lies between the reciprocal normal section curves and divides the angle between the normal sections in the ratio of 1/3 to 2/3.

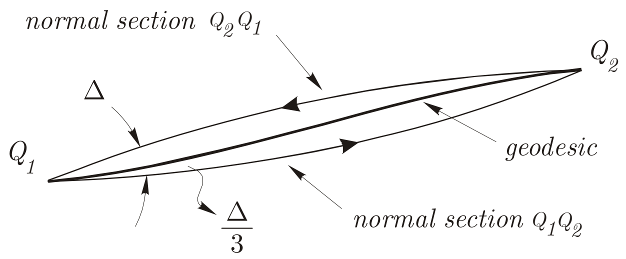

Figure 5. Geodesic curve between normal sections

In equation (4) the correction from the normal section to the geodesic, the term in braces { }, will be in radians. The conversion from radians to seconds of arc is:

$$\text{seconds of arc} = \text{radians} \times \frac{180}{\pi} \times 3600\,.$$

The correction given by equation (4) is accurate to at least 0.001" for lines up to 150 km in length on the ellipsoid. At 1500 km, the correction is approximately 7" and the formula is accurate to about 0.6". At greater distances the accuracy of the formula deteriorates and other methods should be used to determine the correction.

The difference between the length of the geodesic and either of the normal sections seldom attains 1 mm at 1500 km and can be ignored for all practical purposes (NMC 1985).

REFERENCES

Krakiwsky, E.J. and Thomson, D.B., 1974. *Geodetic Position Computations,* 1990 re-print, Department of Surveying Engineering, University of Calgary, Calgary, Alberta, Canada.

NMC, 1985. *The Australian Geodetic Datum Technical Manual,* Special Publication 10, National Mapping Council of Australia, Canberra, 1985

RMIT Lecture Notes, *Skew-normal Correction to Geodetic Directions on an Ellipsoid,* 8 pages.

Vanicek, P. and Krakiwsky, E., 1986. *Geodesy: The Concepts,* North-Holland, Amsterdam.

DETERMINATION OF ELLIPSOIDAL HEIGHT DIFFERENCES FROM VERTICAL CIRCLE THEODOLITE OBSERVATIONS AND SLOPE DISTANCES

To reduce a chord distance (or slope distance) between points P_1 and P_2 on the Earth's terrestrial surface to a distance s on the ellipsoid between the projections Q_1 and Q_2, the ellipsoidal heights h_1 and h_2 at P_1 and P_2 must be known. Ellipsoidal height differences $\Delta h = h_2 - h_1$ can be determined from vertical circle theodolite observations and measured slope distances and ellipsoidal heights of successive points of a traverse determined from a known starting height. Two formulae can be used; one using the chord distance D and the other using the geodesic distance s

Chord distance (or slope distance) D known

$$\Delta h_{12} = h_2 - h_1 = D\cos z_1 + \frac{D^2}{2R_\alpha}(1-2k)\sin^2 z_1 + i_1 - g_2 \tag{5}$$

Geodesic distance s known

$$\Delta h_{12} = h_2 - h_1 = \left(1 + \frac{h_1}{R_\alpha}\right)\left\{\frac{s}{\tan z_1} + \frac{s^2}{2R_\alpha}(1-2k)\right\} + i_1 - g_2 \tag{6}$$

where

D is the chord distance (or slope distance) between points 1 and 2

h_1, h_2 are the ellipsoidal heights of stations 1 and 2

$\Delta h = h_2 - h_1$ is the ellipsoidal height difference

g_2 is the height of target at point 2

i_1 is the height of instrument at point 1

k is the coefficient of refraction

s is the geodesic distance between points 1 and 2

z_1 is the zenith distance, measured with respect to the normal at point 1, to point 2

$R_\alpha = \dfrac{\rho_1 \nu_1}{\rho_1 \sin^2 \alpha_{12} + \nu_1 \cos^2 \alpha_{12}}$ is the radius of curvature at point 1 of the normal section in the direction α_{12}

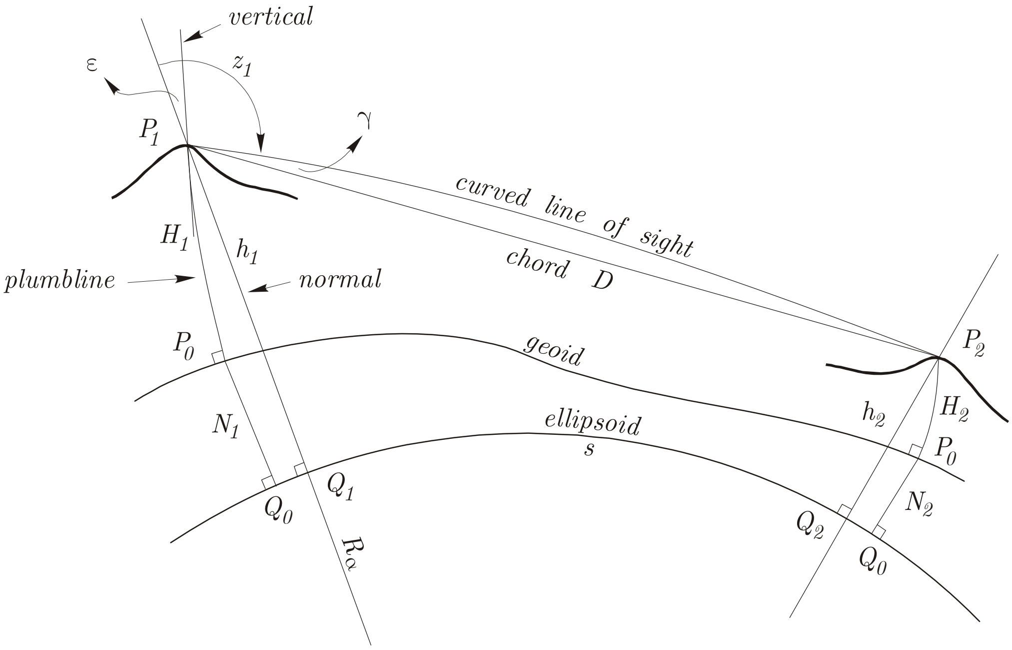

Figure 6. Zenith distance measurements and height differences

In Figure 6, P_1 and P_2 are at ellipsoidal heights h_1 and h_2 above the ellipsoid. Ellipsoidal heights are distances measured along the normal, $h_1 = P_1Q_1$ and $h_2 = P_2Q_2$. Orthometric heights H_1 and H_2 are distances measured along the curved plumbline from the geoid to points P_1 and P_2. In Australia, the Australian Height Datum (AHD) is a gently undulating surface approximating Mean Seal Level around the Australian coast and is regarded as a reasonable approximation of the geoid across Australia. Hence AHD values can be accepted as orthometric heights for computational purposes. N_1, N_2 are geoid-ellipsoid separations measured along ellipsoid normals Q_0P_0. They cannot be determined directly but may be derived from geoid model such as AUSGeoid98. The connection between ellipsoidal and orthometric heights is given by the equation $h = H + N$ but from Figure 6 it is clear that these quantities are not directly connected since the plumbline is a 3D space curve and

P, Q, P_0 and Q_0 are not collinear. Nevertheless, for all practical purposes $h = H + N$ can be regarded as an exact relationship and AHD values can be substituted for H (Featherstone & Rüeger 2000).

In equations (5) and (6), and Figure 6 z_1 is the zenith distance, measured with respect to the normal at point 1, to the target at point 2, and theodolite vertical circle observations (made with respect to the vertical) should first be corrected for gravimetric effects (the value $\varepsilon = \xi \cos \alpha + \eta \sin \alpha$) before using in the formulae; see equation (2).

In Figure 6, the line of sight P_1P_2 is curved or refracted by the Earth's atmosphere and γ is a small angle, known as the angle of refraction, between the curved line of sight and the chord *D*. By letting $\gamma = k\theta = ks/R_\alpha$ where *k* is the coefficient of refraction and θ is the angle subtended by an arc length *s* at the centre of a circular arc of radius R_α. The effects of refraction are allowed for in the development of equations (5) and (6) but the value of *k* cannot be determined with any degree of certainty, since it is known to vary according to the time of day, the atmospheric conditions, the length of line and the direction of the line of sight. Using *reciprocal verticals,* i.e., vertical circle observations observed simultaneously from both ends of the line, can eliminate this uncertainty assuming that the coefficient of refraction will be the same (or nearly so) at both ends of the line. This assumption is valid if the Earth's atmosphere is stable and evenly heated and from practice it is well known that reciprocal verticals will only yield reliable height differences if the observing conditions are reasonable and the observations are made between the hours of 11 am and 3 pm. When using equations (5) or (6) a value of $k = 0.07$ for *average* conditions is used and height differences Δh_{12} and Δh_{21} computed from observations at both ends; the mean result assumed free of uncertainty in the value of *k.* For a complete treatment of the effects of atmospheric refraction on vertical angles the reader is directed to Bomford (1980, pp. 228-243)

It should be noted that some authors define the coefficient of refraction as a ratio of radii of curvature, $k' = R_\alpha/\sigma$ where σ is the radius of curvature of the curved line of sight. This leads to $\gamma \approx k'/(2\theta)$ or $k' = 2k$. Equations (5) and (6) could contain the term $(1 - k')$ rather than $(1 - 2k)$.

Equations (5) and (6) are not exact relationships and certain practical assumptions are made in their development (RMIT 1984). Similar equations are derived in Rüeger (1990, pp. 108-114).

REFERENCES

Bomford, G., 1980. *Geodesy,* 4th edn, Clarendon Press, Oxford

Featherstone, W.E. and Rüeger, J.M., 2000. 'The importance of using deflections of the vertical for reduction of survey data to a geocentric datum', *The Trans Tasman Surveyor,* Vol. 1, No. 3, pp. 46-61, December 2000. See also "Erratum" in *The Australian Surveyor,* Vol. 47, No. 1, p.7, June 2002.

Rüeger, J.M., 1990. *Electronic Distance Measurement – An Introduction,* 3rd edn, Springer-Verlag, Berlin.

RMIT Lecture Notes, *Heights by Vertical Angles,* February 1984, 15 pages.

REDUCTION OF DISTANCES TO THE ELLIPSOID

To reduce a chord distance D between points P_1 and P_2 on the Earth's terrestrial surface to a distance s on the ellipsoid between the projections Q_1 and Q_2, the ellipsoidal heights h_1 and h_2 at P_1 and P_2 are assumed known and $\Delta h = h_2 - h_1$. A circular arc chord distance c is computed, on the assumption that a circular arc of radius R_α is a close approximation of the normal section elliptical arc between Q_1 and Q_2. The geodesic distance s is regarded as equal to the distance along the circular arc.

$$c = R_\alpha \sqrt{\frac{D^2 - (\Delta h)^2}{(R_\alpha + h_1)(R_\alpha + h_2)}} \tag{7}$$

$$s = 2R_\alpha \sin^{-1}\left(\frac{c}{2R_\alpha}\right) \tag{8}$$

where

D is the chord distance (or slope distance) between points 1 and 2

h_1, h_2 are the ellipsoidal heights of stations 1 and 2

$\Delta h = h_2 - h_1$ is the ellipsoidal height difference

s is arc length of a circular arc of radius R_α

c is the chord distance of a circular arc of radius R_α

$R_\alpha = \dfrac{\rho_1 \nu_1}{\rho_1 \sin^2 \alpha_{12} + \nu_1 \cos^2 \alpha_{12}}$ is the radius of curvature at point 1 of the normal section in the direction α_{12}

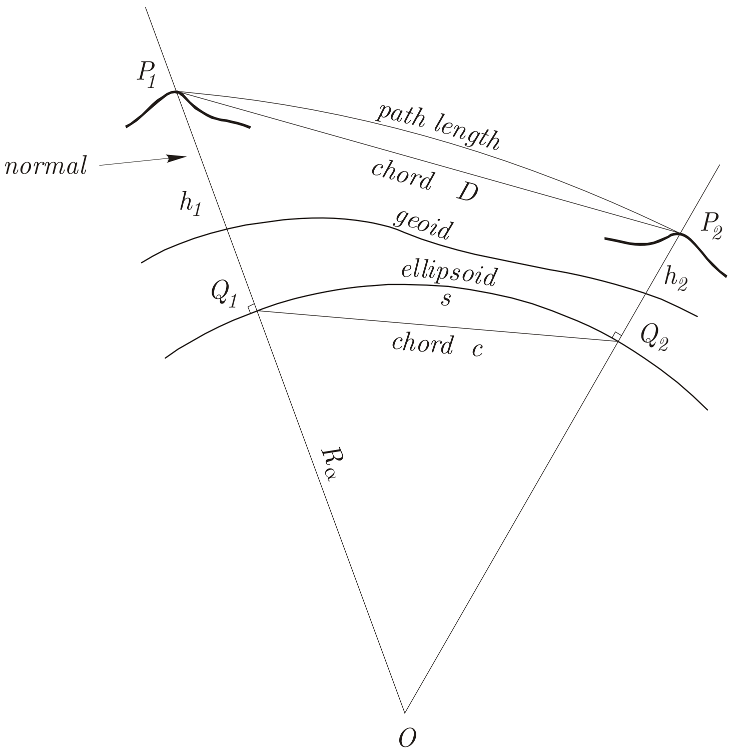

Figure 7. The geometry of the reduction of measured slope distances

Figure 7 shows P_1 and P_2 at ellipsoidal heights $h_1 = P_1Q_1$ and $h_2 = P_2Q_2$ above the ellipsoid and $\Delta h = h_2 - h_1$ is the ellipsoidal height difference. The chord distance $D = P_1P_2$ is the measured slope distance. The actual distance measured by an EDM is the length of a curved electromagnetic wave path called the raw-distance. Atmospheric corrections are applied and a path curvature correction also applied to compensate for the difference between the *mean* atmospheric conditions from observations at both ends of the line and the *average* atmospheric conditions over the length of the line. The path length = raw-distance + atmospheric correction + path curvature correction, is shown on Figure 7 as the curved path P_1P_2. A correction is applied to the curved path length to give the chord distance D. It is assumed, in these notes that all measured slope distances have been reduced to chord distances D by the application of suitable corrections.

In Figure 7, the normal section ellipsoidal arc between Q_1 and Q_2 is closely approximated by a circular arc of radius R_α with a centre at O and the ellipsoidal chord c can be assumed to be a chord of a circular arc. The geodesic distance s is assumed, for all practical purposes, to be equal to the arc length of a circular curve of radius R_α. This is a reasonable assumption since for two points $\phi_1 = -38^\circ$, $\lambda_1 = 145^\circ$, $h_1 = 1000$ m and $\phi_2 = -37^\circ$, $\lambda_2 = 146^\circ$, $h_1 = 1000$ m on the GRS80 ellipsoid $\left(a = 6378160, f = 1/298.257222101\right)$ the azimuth and geodesic distance are $\alpha_{12} = 38^\circ\,51'\,01.5705''$ and $s = 141903.347$ m . The geodesic distance computed using equations (7) and (8) is 141903.348 m, a difference of 0.001 m.

EXAMPLE TRAVERSE REDUCTIONS

Table 2 shows a set of observations made between the stations *Smeaton, Buninyong, Flinders Peak, Bellarine, Arthur's Seat* and *Bass* and Figure 8 shows a diagram of the traverse. *Smeaton, Buninyong, Arthur's Seat* and *Bass* are fixed stations and the coordinates of *Flinders Peak* and *Bellarine* are required. No corrections have been applied to the theodolite observations that are assumed to be the means of sets of horizontal directions (H) and vertical circle observations (V). The slope distances are assumed to be chord distances between observing stations.

Station		Theodolite Observations	Slope distance	Heights	
AT	TO			Inst.	Target
Buninyong	Smeaton	H 0°00´00˝ V 90°15´02.92˝		1.650	1.650
Buninyong	Flinders Peak	H 119°47´10.10˝ V 90°37´42.36˝	54978.184	1.650	1.585
Flinders Peak	Buninyong	H 0°00´00˝ V 89°47´52.25˝		1.710	1.610
Flinders Peak	Bellarine	H 196°43´49.42˝ V 90°32´35.99˝	27661.033	1.710	1.685
Bellarine	Flinders Peak	H 0°00´00˝ V 89°40´12.54˝		1.660	1.760
Bellarine	Arthur's Seat	H 163°45´32.30˝ V 89°51´41.91˝	37176.908	1.660	1.590
Arthur's Seat	Bellarine	H 0°00´00˝ V 90°25´24.13˝		1.680	1.775
Arthur's Seat	Bass	H 158°34´37.43˝ V 90°15´57.84˝		1.680	1.680

Table 2. Observations of the *Buninyong-Arthur's Seat* traverse

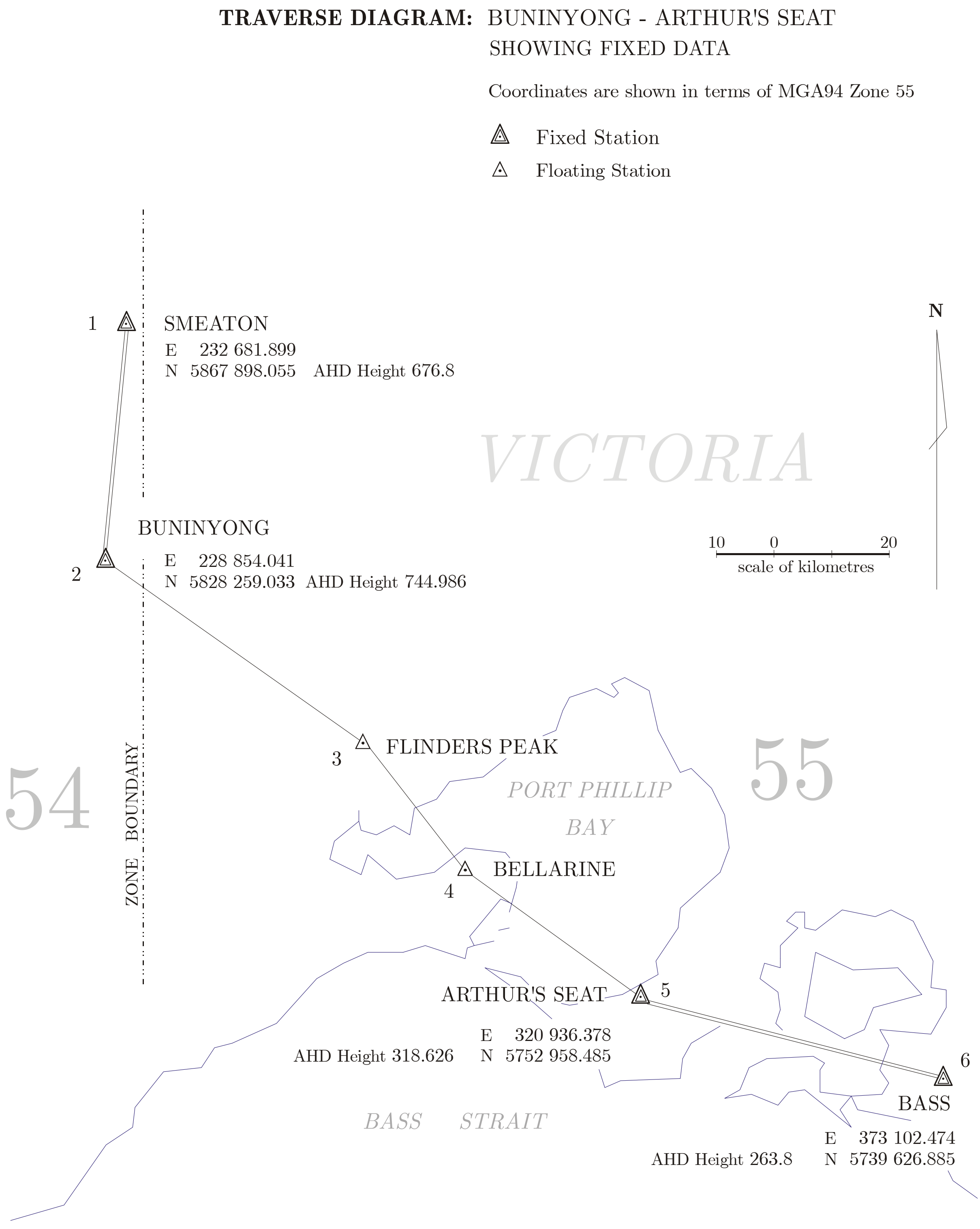

Figure 8. *Buninyong-Arthur's Seat* traverse

STEP 1 Computation of deflection of the vertical components ξ and η and geoid-ellipsoid separations (N-values)

To apply gravimetric corrections to the theodolite observations, the deflection components ξ and η need to be computed for each traverse station. This can be done using AUSGeoid98 available at the Geoscience Australia website (http://www.ga.gov.au/) following the links Geodesy & GPS, then AUSGeoid and finally Compute an N value on-line. The entry values required are latitude and longitude and the computed output is an N-value (geoid-ellipsoid separation in metres) and deflections (ξ and η in seconds of arc).

The approximate latitudes and longitudes of the traverse stations can be computed using the observed directions and slope distances and two Microsoft® Excel spreadsheets available at the Geoscience Australia website by following the links to Geodetic Calculations then Calculate Bearing Distance from Latitude Longitude. At this web page two Microsoft Excel spreadsheets are available:

(i) Vincenty.xls will compute the *direct* case on the ellipsoid (given latitude and longitude of point 1 and the azimuth and geodesic distance to point 2, compute latitude and longitude of point 2) and the *inverse* case (given the latitudes and longitudes of points 1 and 2 compute the azimuth and geodesic distance between them) and

(ii) Redfearn.xls will convert GDA latitudes and longitudes on the ellipsoid to MGA East and North coordinates on a Universal Transverse Mercator (UTM) projection with point scale factor k and grid convergence γ and vice-versa.

1.1 Use Redfearn.xls to convert the Map Grid Australia (MGA94) Zone 55 grid coordinates of the fixed stations *Smeaton, Buninyong, Arthur's Seat* and *Bass* to Geocentric Datum of Australia (GDA94) latitudes and longitudes. The reference ellipsoid of GDA94 is the reference ellipsoid of the Geodetic Reference System 1980 (GRS80) $a = 6378137 \text{ m}, f = 1/298.257222101$. The computed latitudes and longitudes are shown in Table 3.

	Station	East	North	Latitude	Longitude
1	Smeaton	232681.899	5867898.055	-37°17´49.7306˝	143°59´03.1691˝
2	Buninyong	228854.041	5828259.033	-37°39´10.1563˝	143°55´35.3835˝
5	Arthur's Seat	320936.378	5752958.485	-38°21´13.1263˝	144°57´02.5549˝
6	Bass	373102.474	5739626.885	-38°28´57.6104˝	145°32´42.3666˝

Table 3. MGA Zone 55 and GDA coordinates of the fixed stations of the traverse.

1.2 Use Vincenty.xls to compute the geodesic azimuth and geodesic distance of the lines *Smeaton-Buninyong* and *Arthur's Seat-Bass.* One will be the starting azimuth and the other can be used as a check on the angular misclose of the traverse.

Line	Station	Azimuth α	geodesic distance s
2-1	Buninyong-Smeaton	7°23´13.037˝	39803.797
5-6	Arthur's Seat-Bass	105°36´33.043˝	53848.539

Table 4. Geodesic azimuths and distances of fixed lines

1.3 Use the fixed azimuth of the line *Buninyong-Smeaton* (as the starting azimuth) and the observed directions and slope distance (Table 2) to obtain approximations of the geodesic azimuth and geodesic distance of the traverse line *Buninyong-Flinders Peak.* Use these values ($\alpha \approx 127^\circ\, 10'\, 23.137'', s \approx 54978.184$ m) and the GDA coordinates of *Buninyong* in Vincenty.xls (Direct Solution) to compute the GDA coordinates of *Flinders Peak.* The direct solution in Vincenty.xls will also give the reverse azimuth *Flinders Peak-Buninyong* $306^\circ\, 52'\, 03.313''$, which is the starting azimuth for the next traverse line *Flinders Peak-Bellarine.* This procedure is repeated for each traverse line and Table 5 shows the computed results and the angular misclose of the traverse ($0.824''$).

Observed traverse lines	Azimuth	Distance	Point
			Buninyong (Fixed) -37°39´10.1563˝ lat 143°55´35.3835˝ long
Buninyong-Smeaton (Fixed) Buninyong-Flinders Peak (obs)	7°23´13.037˝ + 119°47´10.10˝ = 127°10´23.137˝	54978.184	Flinders Peak (comp) -37°57´03.8222˝ 144°25´29.7304˝
Flinders Peak-Buninyong (comp) Flinders Peak-Bellarine (obs)	306°52´03.313˝ + 196°43´49.42˝ = 143°35´52.733˝	27661.033	Bellarine (comp) -38°09´05.3661˝ 144°36´43.9373˝
Bellarine-Flinders Peak (comp) Bellarine-Arthur's Seat (obs)	323°28´57.174˝ + 163°45´32.30˝ = 127°14´29.474˝	37176.908	Arthur's Seat (comp) -38°21´13.2868˝ 144°57´02.8792˝
Arthur's Seat-Bellarine (comp) Arthur's Seat-Bass (obs)	307°01´54.789˝ + 158°34´37.43˝ = 105°36´32.219˝		Arthur's Seat (Fixed) -38°21´13.1263˝ 144°57´02.5549˝
Arthur's Seat-Bass (Fixed) Misclose (Fixed-Observed)	105°36´33.043˝ - 105°36´32.219˝ = 0°00´00.824˝		Misclose (Fixed-comp) -0.1605˝ lat -0.3243˝ long

Table 5. Fixed and approximate GDA coordinates of traverse stations.

1.4 The geoid-ellipsoid separations (*N*-values) and deflection components ξ and η are computed using AUSGeoid98 available at the Geoscience Australia website (http://www.ga.gov.au/) following the links Geodesy & GPS, then AUSGeoid and finally Compute an N value on-line. The entry values required are latitude and longitude (see Table 5) and the computed values are shown in Table 6.

	Station	Latitude	Longitude	N	ξ	η
1	Smeaton	-37°17´50″	143°59´03″	5.705 m	+0.076″	-6.554″
2	Buninyong	-37°39´10″	143°55´35″	4.869 m	-5.982″	-3.817″
3	Flinders Peak	-37°57´04″	144°25´30″	3.748 m	-9.878″	-1.606″
4	Bellarine	-38°09´05″	144°36´44″	2.979 m	-8.029″	-2.453″
5	Arthur's Seat	-38°21´13″	144°57´03″	3.095 m	-2.557″	-9.242″
6	Bass	-38°28´58″	145°32´42″	3.904 m	-5.169″	-4.428″

Table 6. Geoid-ellipsoid separations N and deflection components ξ and η.

The output from AUSGeoid98 rounds the latitudes and longitudes to the nearest second of arc and ξ and η are components of the deflection of the vertical at the geoid.

STEP 2 Computation of gravimetric corrections to theodolite observations

Using equations (1) and (2), the azimuths of the traverse lines from Table 5, the deflection components ξ and η from Table 6 and the theodolite observations from Table 2, corrections to the theodolite observations are computed and shown in Table 7. The corrected theodolite observations (H is horizontal direction and V is zenith distance) are quasi-measurements made with a theodolite at P whose rotational axis is coincident with the normal at P and the horizontal plane of the theodolite is the geodetic horizon plane at P (the geodetic horizon plane at P is a plane parallel to the plane tangential to the ellipsoid at Q).

Line	Azimuth α	Theodolite observations	corr'ns	Theodolite observations corrected to the normal
Buninyong-Smeaton	7°23´13.037″	H 0°00´00″ V 90°15´02.92″	-0.020″ -6.423″	H 0°00´00″ V 90°14´56.497″
Buninyong-Flinders Peak	127°10´23.137″	H 119°47´10.10″ V 90°37´42.36″	-0.027″ +0.573″	H 119°47´10.093″ V 90°37´42.933″
Flinders Peak-Buninyong	306°52´03.313″	H 0°00´00″ V 89°47´52.25″	-0.024″ -4.642″	H 0°00´00″ V 89°47´47.608″
Flinders Peak-Bellarine	143°35´52.733″	H 196°43´49.42″ V 90°32´35.99″	-0.043″ +6.997″	H 196°43´49.401″ V 90°32´42.987″
Bellarine-Flinders Peak	323°28´57.174″	H 0°00´00″ V 89°40´12.54″	-0.016″ -4.993″	H 0°00´00″ V 89°40´07.547″
Bellarine-Arthur's Seat	127°14´29.474″	H 163°45´32.30″ V 89°51´41.91″	+0.012″ +2.906″	H 163°45´32.328″ V 89°51´44.816″
Arthur's Seat-Bellarine	307°01´54.789″	H 0°00´00″ V 90°25´24.13″	-0.026″ +5.838″	H 0°00´00″ V 90°25´29.968″
Arthur's Seat-Bass	105°36´33.043″	H 158°34´37.43″ V 90°15´57.84″	+0.000″ -8.213″	H 158°34´37.456″ V 90°15´49.627″

Table 7. Theodolite observations, corrected to ellipsoid normals, for the *Buninyong-Arthur's Seat* traverse.

STEP 3 Computation of ellipsoidal heights h and AHD heights by vertical angles

3.1 Ellipsoidal height differences Δh between traverse stations are computed using equation (5) with zenith distances (corrected to the normal) from Table 7and slope distances D from Table 2. R_α, the radii of curvature in the azimuths of the traverse lines, are required and to compute these values, the radii of curvature and ν (meridian and the prime vertical respectively) are required. These can be computed using Redfearn.xls with the approximate latitudes and longitudes of the traverse stations from Table 6 that are sufficiently accurate to give the radii to the nearest metre. Table 8 shows the radii of curvature ρ and ν.

	Station	Latitude	Longitude	ρ	ν
1	Smeaton	-37°17´50″	143°59´03″	6358870	6385990
2	Buninyong	-37°39´10″	143°55´35″	6359254	6386119
3	Flinders Peak	-37°57´04″	144°25´30″	6359577	6386227
4	Bellarine	-38°09´05″	144°36´44″	6359794	6386299
5	Arthur's Seat	-38°21´13″	144°57´03″	6360014	6386373
6	Bass	-38°28´58″	145°32´42″	6360154	6386420

Table 8. Radii of curvature of the ellipsoid (nearest metre).

3.2 Using the values for ρ and ν from Table 8 and the azimuths of the traverse lines from Table 5, rounded to the nearest second of arc, the radii of curvature in the azimuths of the traverse lines are computed from

$$R_\alpha = \frac{\rho\nu}{\rho\sin^2\alpha + \nu\cos^2\alpha} \tag{9}$$

Note that the value of R_α is different at the ends of the same line.

Line	Azimuth	R_α
Buninyong-Flinders Peak	127°10´23″	6376285
Flinders Peak-Buninyong	306°52´03″	6376608
Flinders Peak-Bellarine	143°35´53″	6368937
Bellarine-Flinders Peak	323°28´57″	6369154
Bellarine-Arthur's Seat	127°14´30″	6376566
Arthur's Seat-Bellarine	307°01´55″	6376788

Table 9. Radii of curvature of the ellipsoid in azimuths of traverse lines (nearest metre)

3.3 The ellipsoidal height differences Δh, computed using equation (5) with $k = 0.07$ as the coefficient of refraction, are shown in Table 10 with mean results for each traverse line.

Line	Zenith distance z	Slope distance D	R_α	Heights		Height difference Δh
				Inst.	Target	
Buninyong-Flinders Peak	90°37´42.933″	54978.184	6376285	1.650	1.585	-399.277
Flinders Peak-Buninyong	89°47´47.608″		6376608	1.710	1.610	+399.136
					Mean	-399.207
Flinders Peak-Bellarine	90°32´42.987″	27661.033	6368937	1.710	1.685	-211.563
Bellarine-Flinders Peak	89°40´07.547″		6369154	1.660	1.760	+211.467
					Mean	-211.515
Bellarine-Arthur's Seat	89°51´44.816″	37176.908	6376566	1.660	1.590	+182.523
Arthur's Seat-Bellarine	90°25´29.968″		6376788	1.680	1.775	-182.658
					Mean	+182.590

Table 10. Computed ellipsoidal height differences and mean height differences for the *Buninyong-Arthur's Seat* traverse.

3.4 Ellipsoidal heights of the traverse stations are computed by adding the mean height differences From Table 10 to the starting value at *Buninyong:*

$h = H + N = 744.986 + 4.869 = 749.855$

	Station	Fixed AHD Height H	N	Ellipsoidal Height h	Computed AHD Height H
1	Smeaton	676.8	5.705	682.505	
2	Buninyong	744.986	4.869	749.855	
3	Flinders Peak		3.748	749.855 - 399.207 = 350.648	346.900
4	Bellarine		2.979	350.648 - 211.515 = 139.133	136.154
5	Arthur's Seat	318.626	3.095	139.133 + 182.590 = 321.723	318.628
6	Bass	263.8	3.904	267.704	

Table 11. Ellipsoidal and AHD heights for the *Buninyong-Arthur's Seat* traverse.

Note that the height misclose, the difference between the Fixed and Computed AHD Heights at Arthur's Seat, is not representative of height closures obtained from verticals.

STEP 4 Computation of geodesic distances

The geodesic distances s are computed by first computing ellipsoidal chord distances c using equation (7) and then using the chord distances c in equation (8). Table 12 shows the ellipsoidal chord distances c and the geodesic distances s.

Line	Slope distances D	Height diff.	R_α	Ellipsoidal heights h_1, h_2	chord distance c	geodesic distance s
Buninyong-Flinders Peak	54978.184	-399.207	6376285	749.855 350.648	54971.991	54972.161
Flinders Peak-Bellarine	27661.033	-211.515	6368937	350.648 139.133	27659.161	27659.183
Bellarine-Arthur's Seat	37176.908	+182.590	6376566	139.133 321.723	37175.116	37175.169

Table 12. Computed geodesic distances for the *Buninyong-Arthur's Seat* traverse.

STEP 5 Skew-normal corrections

The skew-normal corrections to the observed theodolite directions are computed using equation (3) with the ellipsoidal heights h_2 of the stations from Table 11 (rounded to the nearest metre), the mean radius of curvature $\rho_m = (\rho_1 + \rho_2)/2$ from Table 8 (rounded to the nearest metre), approximate azimuths from Table 5 and latitudes ϕ_2 from Table 6 (nearest second of arc). The skew-normal corrections are shown in Table 13.

Line	h_2	ρ_m	Azimuth (Table 5)	Latitude (Table 6)	corr'n
1 Buninyong- 2 Smeaton	683	6359062	7°23´13.037˝	-37°17´50˝	+0.012˝
1 Buninyong- 2 Flinders Peak	351	6359416	127°10´23.137˝	-37°57´04˝	-0.023˝
1 Flinders Peak- 2 Buninyong	750	6359416	306°52´03.313˝	-37°39´10˝	-0.049˝
1 Flinders Peak- 2 Bellarine	139	6359686	143°35´52.733˝	-38°09´05˝	-0.009˝
1 Bellarine- 2 Flinders Peak	351	6359686	323°28´57.174˝	-37°57´04˝	-0.023˝
1 Bellarine- 2 Arthur's Seat	322	6359904	127°14´29.474˝	-38°21´13˝	-0.021˝
1 Arthur's Seat- 2 Bellarine	139	6359904	307°01´54.789˝	-38°09´05˝	-0.009˝
1 Arthur's Seat- 2 Bass	268	6360084	105°36´33.043˝	-38°28´58˝	-0.009˝

Table 13. Skew-normal corrections to theodolite directions for the *Buninyong-Arthur's Seat* traverse.

STEP 6 Correction from the normal section to the geodesic

The corrections from the observed normal section directions to the geodesic directions are computed using equation (4) with the geodesic distances s from Tables 4 and 12, the mean radius of curvature $\nu_m = (\nu_1 + \nu_2)/2$ from Table 8 (rounded to the nearest metre), approximate azimuths from Table 5 and mean latitudes $\phi_m = (\phi_1 + \phi_2)/2$ from Table 6 (nearest second of arc). The corrections from the observed normal section directions to the geodesic directions are shown in Table 14.

Line	s	ν_m	Azimuth α_{12} (Table 5)	Latitude ϕ_m (Table 6)	corr'n
1 Buninyong- 2 Smeaton	38903.797	6386055	7°23´13.037″	-37°28´30″	-0.001″
1 Buninyong- 2 Flinders Peak	54972.161	6386173	127°10´23.137″	-37°48´07″	+0.005″
1 Flinders Peak- 2 Buninyong	54972.161	6386173	306°52´03.313″	-37°48´07″	+0.005″
1 Flinders Peak- 2 Bellarine	27659.183	6386263	143°35´52.733″	-38°03´05″	+0.001″
1 Bellarine- 2 Flinders Peak	27659.183	6386263	323°28´57.174″	-38°03´05″	+0.001″
1 Bellarine- 2 Arthur's Seat	37175.169	6386336	127°14´29.474″	-38°15´09″	+0.002″
1 Arthur's Seat- 2 Bellarine	37175.169	6386336	307°01´54.789″	-38°15´09″	+0.002″
1 Arthur's Seat- 2 Bass	53849.539	6386396	105°36´33.043″	-38°25´06″	+0.003″

Table 14. Corrections from normal section directions to geodesic directions.

STEP 7 Calculation of geodesic directions

The directions of the geodesics on the ellipsoid (geodesic directions) are obtained by adding the skew-normal corrections (Table 13) and the corrections from the normal section to the geodesic (Table 14) to the theodolite directions corrected to the normal (Table 7). Table 15 shows the geodesic directions reduced to $0^\circ\,00'\,00''$ on the backsight.

Line	Theodolite directions corrected to the normal	corrections		Geodesic directions
		skew-normal	geodesic	
Buninyong-Smeaton	0°00´00″	+0.012″	-0.001″	0°00´00″
Buninyong-Flinders Peak	119°47´10.093″	-0.023″	+0.005″	119°47´10.064″
Flinders Peak-Buninyong	0°00´00″	-0.049″	+0.005″	0°00´00″
Flinders Peak-Bellarine	196°43´49.401″	-0.009″	+0.001″	196°43´49.437″
Bellarine-Flinders Peak	0°00´00″	-0.023″	+0.001″	0°00´00″
Bellarine-Arthur's Seat	163°45´32.328″	-0.021″	+0.002″	163°45´32.331″
Arthur's Seat-Bellarine	0°00´00″	-0.009″	+0.002″	0°00´00″
Arthur's Seat-Bass	158°34´37.456″	-0.009″	+0.003″	158°34´37.457″

Table 15. Geodesic directions, for the *Buninyong-Arthur's Seat* traverse.

STEP 8 Traverse observations reduced to the ellipsoid

Table 16 shows the set of measurements on the ellipsoid that are used to compute the Map Grid Australia (MGA94) coordinates of the traverse stations *Flinders Peak* and *Bellarine*.

	Station / Geodesic Distance	Geodesic angle	
1	Smeaton		
2	Buninyong	1-2-3	119°47´10.06″
	(2-3) 54972.161 m		
3	Flinders Peak	2-3-4	196°43´49.44″
	(3-4) 27659.183 m		
4	Berllarine	3-4-5	163°45´32.33″
	(4-5) 37175.169 m		
5	Arthur's Seat	4-5-6	158°34´37.46″
6	Bass		

Table 16. Geodesic distances and angles of the *Buninyong-Arthur's Seat* traverse

COMMENTARY ON THE REDUCTION OF OBSERVATIONS TO THE ELLIPSOID

The reduction of traverse observations to the ellipsoid set out on the previous pages may be regarded as exact for all practical purposes for traverse lines up to 55 km in length. For shorter traverse lines certain corrections may be ignored without any practical loss of accuracy, e.g., inspection of Tables 7 and 15 show that corrections to observed horizontal directions (gravimetric, skew-normal and geodesic) do not exceed 0.03" for the *Buninyong-Arthur's Seat* traverse. It would be a safe to ignore these corrections for traverse lines less than 10 km in length and assume that observed traverse directions are, for practical purposes, directions of geodesics on the ellipsoid.

Gravimetric corrections to vertical circle observations are often ignored in practice. This is justified for the following reasons:

(i) The vertical circle observations are only used to determine ellipsoidal heights for distance reduction, if gravimetric corrections to directions are ignored, and the equation used for computing height differences contains an unknown quantity *k,* the coefficient of refraction.

(ii) To allow for the error in k by assuming a representative value, say $k = 0.07$, only a mean height difference from reciprocal vertical circle observations is regarded as correct.

(iii) If the geoid slope with respect to the ellipsoid is fairly constant along a particular traverse line then the corrections for the deflection of the vertical will be of opposite sign but approximately equal in magnitude and their effects will cancel in the calculation of the mean height difference.

COMPUTATION OF MAP GRID AUSTRALIA (MGA94) COORDINATES

Map Grid Australia (MGA94) coordinates are coordinates related to a grid superimposed over a Universal Transverse Mercator (UTM) projection of latitudes and longitudes related to the Geocentric Datum of Australia (GDA94). The 94 in MGA94 and GDA94 refer to the date of the particular realization of the GDA coordinate set (latitudes and longitudes). The GDA is defined by the size and shape of the reference ellipsoid, the ellipsoid of the Geodetic Reference System 1980 (GRS80) – semi-major axis $a = 6378137$ m, flattening $f = 1/298.257222101$ – and the coordinates of the eight reference stations in the Australian Fiducial Network (AFN). The coordinates of the AFN stations were derived from a global adjustment of geodetic observations and are related to the International Terrestrial Reference Frame of 1992 (ITRF92) at the epoch 1994.0; this effectively fixes the reference ellipsoid at the centre of mass of the Earth with an axis coincident with the Earth's rotational axis as at 1994. The national geodetic data set, consisting of distances, directions and GPS observations between stations in the Australian national geodetic network, was adjusted to fit with the AFN yielding the GDA94 coordinate set.

The UTM projection is a Transverse Mercator (TM) projection of the ellipsoid with defined zone widths and numbering, a central meridian scale factor $k_0 = 0.9996$ and a true origin of coordinates in a zone at the intersection of the equator and the central meridian. MGA94 coordinates are related to a false origin 10,000,000 m south and 500,000 m west of the true origin of a UTM zone.

GDA94 latitudes and longitudes and MGA94 East and North coordinates are related by *Redfearn's* formulae, published by J.C.B Redfearn of the Hydrographic Department of the British Admiralty in the *Empire Survey Review* (now *Survey Review*) in 1948 (Redfearn 1948). Redfearn noted in his five-page paper that: "…formulae of the projection itself have been given by various writers, from Gauss, Schreiber and Jordan to Hristow, Tardi, Lee, Hotine and others – not, it is to be regretted, with complete agreement in all cases." Redfearn's formulae, accurate anywhere within zones of 10°–12° extent in longitude, removed this "disagreement" between previous published formulae and are regarded as the definitive TM formulae. Redfearn provided no method of derivation but mentioned techniques demonstrated by Lee and Hotine in previous issues of the Empire Survey Review. In 1952, the American mathematician Paul D. Thomas published a detailed derivation of the TM formulae in *Conformal Projections in Geodesy and Cartography,* Special Publication No. 251

of the Coast and Geodetic Survey, U.S. Department of Commerce (Thomas 1952); Thomas' work can be regarded as the definitive derivation of the TM formulae.

The conversion between GDA94 and MGA94 coordinates and the computation of azimuths and geodesic distances on the ellipsoid can be achieved using the Microsoft Excel spreadsheets Redfearn.xls and Vincenty.xls available on the Geoscience Australia website (http://www.ga.gov.au) by following the links to Geodetic Calculations then Calculate Bearing Distance from Latitude Longitude. At this web page two Microsoft Excel spreadsheets are available:

(i) Vincenty.xls will compute the *direct* case on the ellipsoid (given latitude and longitude of point 1 and the azimuth and geodesic distance to point 2, compute latitude and longitude of point 2) and the *inverse* case (given the latitudes and longitudes of points 1 and 2 compute the azimuth and geodesic distance between them) and

(ii) Redfearn.xls will convert GDA94 latitudes and longitudes on the ellipsoid to MGA94 East and North coordinates on a Universal Transverse Mercator (UTM) projection with point scale factor k and grid convergence γ and vice-versa.

These two spreadsheets make the computation of traverses a relatively simple matter. There are two methods of computing the MGA94 coordinates of *Flinders Peak* and *Bellarine* in the *Buninyong-Arthur's Seat* traverse. The first method, *Traverse Computation on the Ellipsoid* is a simple direct method, but until recently, has not been practical due to the lack of computational resources. This is no longer the case as any reasonable computer capable of running the Excel spreadsheets Redfearn.xls and Vincenty.xls is adequate (ICSM 2002). The second method, *Traverse Computation on the UTM Map Plane* (NMC 1972, NMC 1985) is a more time consuming indirect method involving iteration. The second method of traverse computation has been, up until now, the only method available to the general practitioner lacking adequate computer resources. Featherstone and Kirby (2002) give an outline of the two methods demonstrating that they give numerically equivalent results (1-3 mm agreement in grid coordinates) and that the second method of computing MGA94 traverse coordinates takes about three times longer than the first method.

TRAVERSE COMPUTATION ON THE ELLIPSOID

This method is direct and simple. Using the reduced traverse observations (geodesic distances and geodesic angles) and coordinates of the fixed stations *Smeaton, Buninyong, Arthur's Seat* and *Bass,* the latitudes and longitudes of the traverse stations are computed using the Excel spreadsheet Vincenty.xls and then converting the latitudes and longitudes to MGA94 East and North coordinates using Redfearn.xls.

The reduced traverse observations are shown in Table 17

	Station / Geodesic Distance	Geodesic angle	
1	Smeaton		
2	Buninyong	1-2-3	119°47´10.06˝
	(2-3) 54972.161 m		
3	Flinders Peak	2-3-4	196°43´49.44˝
	(3-4) 27659.183 m		
4	Berllarine	3-4-5	163°45´32.33˝
	(4-5) 37175.169 m		
5	Arthur's Seat	4-5-6	158°34´37.46˝
6	Bass		

Table 17. Geodesic distances and angles of the *Buninyong-Arthur's Seat* traverse

The traverse diagram is shown in Figure 8 with MGA94 coordinates of the fixed stations in the traverse. It is required to compute the MGA94 coordinates of *Flinders Peak* and *Bellarine.*

STEP 1 Computation of GDA94 coordinates of traverse

1.1 Use Redfearn.xls to convert the MGA94 Zone 55 grid coordinates of the fixed stations *Smeaton, Buninyong, Arthur's Seat* and *Bass* to GDA94 latitudes and longitudes. The reference ellipsoid of GDA94 is the reference ellipsoid of the Geodetic Reference System 1980 (GRS80) $a = 6378137$ m, $f = 1/298.257222101$. The computed latitudes and longitudes are shown in Table 18.

	Station	East	North	Latitude	Longitude
1	Smeaton	232681.899	5867898.055	-37°17´49.7306˝	143°59´03.1691˝
2	Buninyong	228854.041	5828259.033	-37°39´10.1563˝	143°55´35.3835˝
5	Arthur's Seat	320936.378	5752958.485	-38°21´13.1263˝	144°57´02.5549˝
6	Bass	373102.474	5739626.885	-38°28´57.6104˝	145°32´42.3666˝

Table 18. MGA94 Zone 55 and GDA94 coordinates of the fixed stations of the traverse.

1.2 Use Vincenty.xls to compute the geodesic azimuth and geodesic distance of the lines *Smeaton-Buninyong* and *Arthur's Seat-Bass*. One will be the starting azimuth and the other can be used as a check on the angular misclose of the traverse.

Line	Station	Azimuth α	geodesic distance s
2-1	Buninyong-Smeaton	7°23´13.037˝	39803.797
5-6	Arthur's Seat-Bass	105°36´33.043˝	53848.539

Table 19. Geodesic azimuths and distances of fixed lines

1.3 Use the fixed azimuth of the line *Buninyong-Smeaton* (as the starting azimuth) and the reduced geodesic distances and angles (Table 17) to obtain the geodesic azimuth and geodesic distance of the traverse line *Buninyong-Flinders Peak*. Use these values ($\alpha = 127^\circ\, 10'\, 23.097''$ and $s = 54972.161$ m) and the GDA94 coordinates of *Buninyong* in Vincenty.xls (Direct Solution) to compute the GDA94 coordinates of *Flinders Peak*. The direct solution in Vincenty.xls will also give the reverse azimuth *Flinders Peak-Buninyong* $306^\circ\, 52'\, 03.394''$, which is the starting azimuth for the next traverse line *Flinders Peak-Bellarine*. This procedure is repeated for each traverse line and Table 20 shows the computed results and the angular misclose of the traverse ($0.598''$).

Observed traverse lines	Azimuth	Distance	Point
			Buninyong (Fixed) -37°39´10.1563˝ lat 143°55´35.3835˝ long
Buninyong-Smeaton (Fixed) Buninyong-Flinders Peak (obs)	7°23´13.037˝ + 119°47´10.06˝ = 127°10´23.097˝	54972.161	Flinders Peak (comp) -37°57´03.7047˝ 144°25´29.5333˝
Flinders Peak-Buninyong (comp) Flinders Peak-Bellarine (obs)	306°52´03.394˝ + 196°43´49.44˝ = 143°35´52.834˝	27659.183	Bellarine (comp) -38°09´05.2006˝ 144°36´43.6943˝
Bellarine-Flinders Peak (comp) Bellarine-Arthur's Seat (obs)	323°28´57.303˝ + 163°45´32.33˝ = 127°14´29.633˝	37175.169	Arthur's Seat (comp) -38°21´13.0881˝ 144°57´02.5776˝
Arthur's Seat-Bellarine (comp) Arthur's Seat-Bass (obs)	307°01´54.985˝ + 158°34´37.46˝ = 105°36´32.445˝		Arthur's Seat (Fixed) -38°21´13.1263˝ 144°57´02.5549˝
Arthur's Seat-Bass (Fixed) Misclose (Fixed-Observed)	105°36´33.043˝ - 105°36´32.445˝ = 0°00´00.598˝		Misclose (Fixed-comp) +0.0382˝ lat -0.0227˝ long

Table 20. Fixed and computed GDA coordinates of traverse stations.

STEP 2 Computation of MGA94 coordinates

Use Redfearn.xls to convert GDA94 coordinates (latitudes and longitudes) to MGA94 coordinates (East and North). The results are shown in Table 21.

Station	MGA94 coordinates		GDA94 coordinates	
	East	North	Latitude	Longitude
Smeaton (Fixed)	232681.899	5867898.055	-37°17´49.7306˝	143°59´03.1691˝
Buninyong (Fixed)	228854.041	5828259.033	-37°39´10.1563˝	143°55´35.3835˝
Flinders Peak (comp)	272741.501	5796490.264	-37°57´03.7047˝	144°25´29.5333˝
Bellarine (comp)	290769.424	5774687.462	-38°09´05.2006˝	144°36´43.6943˝
Arthur's Seat (comp)	320936.903	5752959.676	-38°21´13.0881˝	144°57´02.5776˝
Arthur's Seat (Fixed)	320936.378	5752958.485	-38°21´13.1263˝	144°57´02.5549˝
Bass (Fixed)	373102.474	5739626.885	-38°28´57.6104˝	145°32´42.3666˝

Table 21. MGA94 Zone 55 and GDA94 coordinates of the traverse.

The linear misclose of the traverse

$$\text{linear misclose} = \sqrt{(\Delta E)^2 + (\Delta N)^2}$$

is computed by obtaining ΔE and ΔN are coordinate differences (Fixed – computed) at *Arthur's Seat:* $\Delta E = -0.525$ m , $\Delta N = -1.191$ m and linear misclose = 1.302 m. The length of the traverse is 119,806.513 m and the accuracy (length/misclose) is approximately 1:92,000. This misclose is quite different from the original traverse computations published in *The Australian Map Grid* Technical Manual, $\Delta E = -0.018$ m , $\Delta N = -0.020$ m (NMC 1972) and the differences are due to the fact that in these notes, the coordinates of the fixed stations are obtained by a transformation of the original AMG66 coordinates to GDA94 using *GDAit*. This transformation is not a uniform translation of coordinates; hence the original traverse misclosures are not linearly connected to the misclosures shown here.

TRAVERSE COMPUTATION ON THE UNIVERSAL TRANSVERSE MERCATOR (UTM) PROJECTION

The *Transverse Mercator* (TM) projection is a conformal projection, i.e., the scale factor at a point is the same in every direction, which means that shape is preserved, although this useful property only applies to differentially small regions of the Earth's surface. Meridians and parallels of the ellipsoid are projected as an orthogonal network of curves, excepting the equator and a central meridian, which are projected as straight lines intersecting at right angles. The intersection of the equator and the central meridian is known as the *true origin* of coordinates and the scale factor along the central meridian is constant.

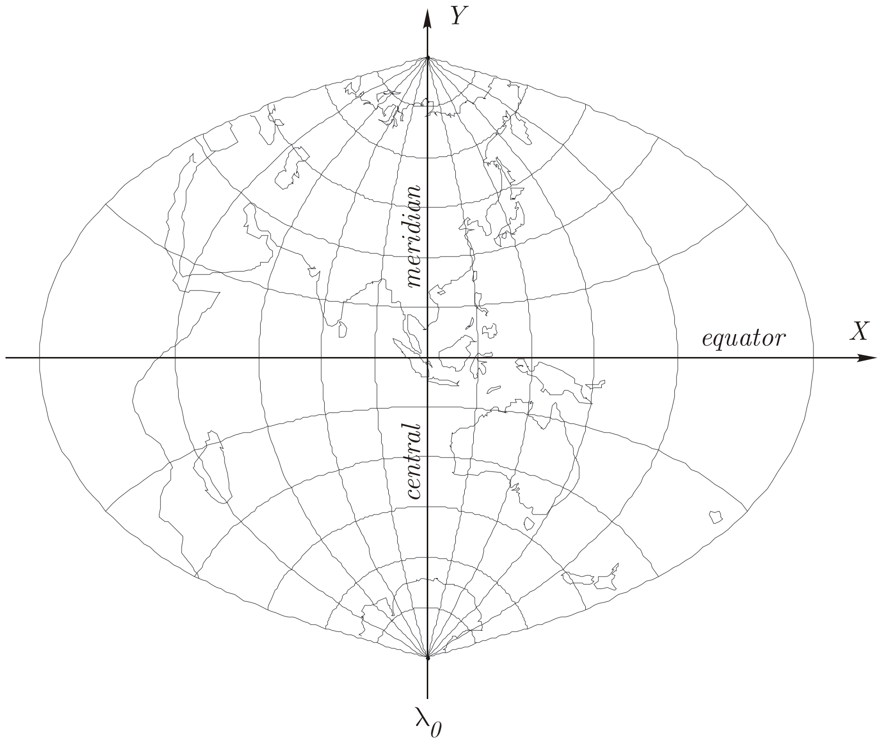

Figure 9. Transverse Mercator projection of part of the ellipsoid. Central meridian $\lambda_0 = 105^\circ$, graticule interval 15°

The TM projection is very useful for mapping regions of the Earth with large extents of latitude, but for areas away from the central meridian, distortions increase rapidly. To limit the effects of distortion, TM projections are usually restricted to small zones of longitude about a central meridian λ_0. The *Universal Transverse Mercator* (UTM) projection is a TM projection of the ellipsoid with defined zone widths of 6° of longitude (3° either side of the central meridian), a zone numbering system (60 zones of 6° width, with zone 1 having a central meridian 177° W and zone 60 having a central meridian of 177° E), a central meridian scale factor $k_0 = 0.9996$ and a true origin of coordinates for each zone at the intersection of the equator and the central meridian. To make coordinates positive quantities, each zone has an origin of East and North coordinates (known as the *false origin*) located 500,000 m west along the equator from the true origin for the northern hemisphere, and 500,000 m west and 10,000,000 m south of the true origin for the southern hemisphere.

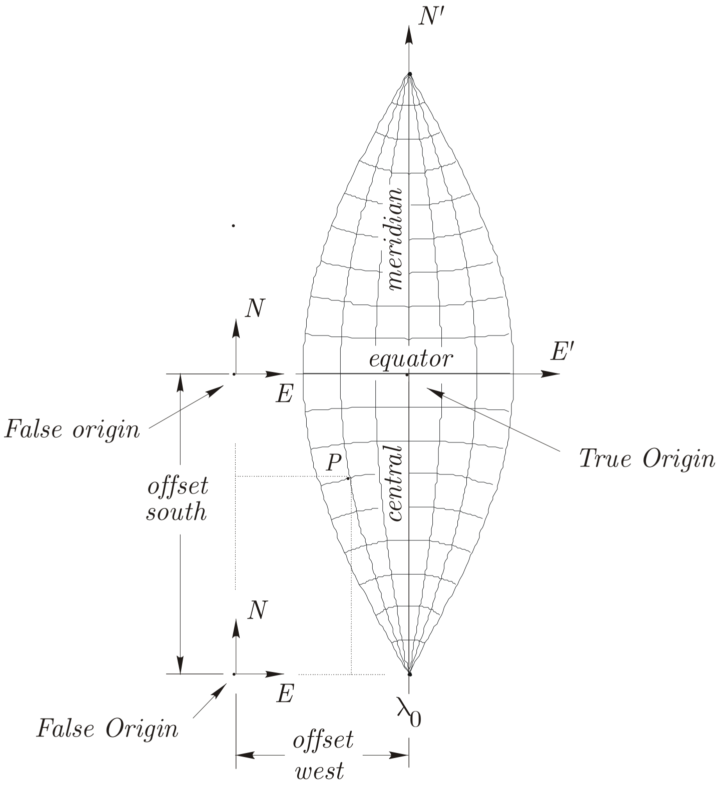

Figure 10 Schematic diagram of a UTM zone showing false origins for the northern and southern hemispheres

Figure 10 shows a schematic diagram of a UTM zone of the Earth. In the southern hemisphere the point P will have negative coordinates E',N' related to the true origin at the intersection of the central meridian and the equator. P has positive E,N coordinates related to the false origin 500,000 m west and 10,000,000 m south of the true origin. True origin and false origin coordinates in the southern hemisphere are related by

$$\begin{aligned} E' &= E - 500{,}000 \\ N' &= N - 10{,}000{,}000 \end{aligned} \tag{10}$$

Figure 11 shows two points P_1 and P_2 on a UTM projection with grid coordinates E_1, N_1 and E_2, N_2. The geodesic s between P_1 and P_2 on the ellipsoid is projected as a curved line concave to the central meridian and shown on the diagram as the *projected geodesic.*
The *plane distance* L is the straight line on the projection and

$$L = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2} \qquad (11)$$

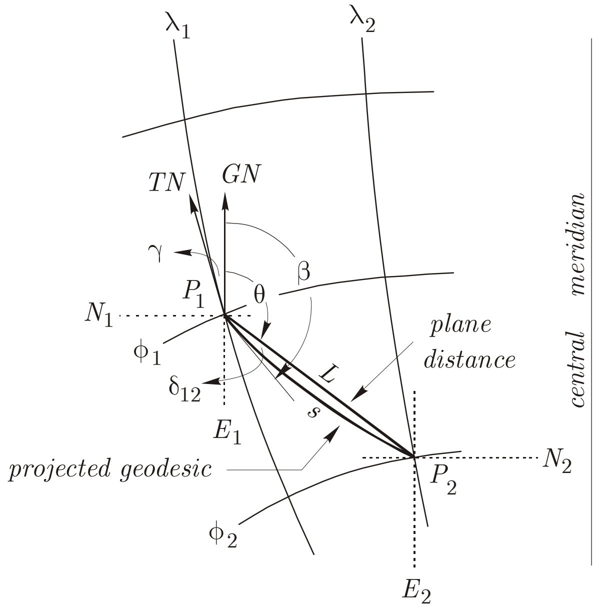

Figure 11. The projected geodesic

The *Line Scale Factor K* is defined as the ratio of plane distance to geodesic distance

$$K = \frac{L}{s} \qquad (12)$$

and the Line Scale Factor can be computed from

$$K = k_0\left[1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{6r_m^2}\left\{1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{36r_m^2}\right\}\right] \qquad (13)$$

where $r_m^2 = \rho\nu k_0^2$ and ρ, ν are computed for $\phi_m = (\phi_1 + \phi_2)/2$. Equation (13) is given in various technical manuals (NMC 1972, NMC 1985 and ICSM 2002) and is regarded as accurate to 0.1 ppm over any 100 km line in a UTM zone. Bomford (1962) compared this formula with others over a known test line and recommended its use. For most practical purposes, the term in braces { } in equation (13) is omitted as its effect is negligible. For a line 100 km in length running north and south on a zone boundary the error in neglecting this term is about 0.25 ppm (NMC 1985).

In Figure 11, *Grid North* (*GN*) is parallel to the direction of the central meridian and *True North* (*TN*) is the direction of the meridian. The angle between True North and Grid North is the *grid convergence* γ. The clockwise angle from Grid North to the tangent to the projected geodesic at P_1 is the *grid bearing* β and the *azimuth* α is the clockwise angle from True North to the tangent to the projected geodesic. Grid bearing and Azimuth are related by

$$\beta = \alpha + \gamma \qquad (14)$$

By convention, in Australia, the grid convergence is a negative quantity west of the central meridian and a positive quantity east of the central meridian.

In Figure 11, the *plane bearing* θ is the clockwise angle from Grid North to the straight line joining P_1 and P_2. The plane bearing is computed from plane trigonometry as

$$\theta = \tan^{-1}\left(\frac{E_2 - E_1}{N_2 - N_1}\right) \tag{15}$$

The small angle between the tangent to the projected geodesic at P_1 and the straight line joining P_1 and P_2 is the *arc-to-chord* correction δ_{12} and is given by

$$\delta_{12} = -\frac{(N_2 - N_1)(E_2' + 2E_1')}{6r_m^2}\left\{1 - \frac{(E_2' + 2E_1')^2}{27r_m^2}\right\} \tag{16}$$

where $r_m^2 = \rho\nu k_0^2$ and ρ, ν are computed for $\phi_m = (\phi_1 + \phi_2)/2$. Equation (16) is given in various technical manuals (NMC 1972, NMC 1985 and ICSM 2002) and is regarded as accurate to about 0.02" over any 100 km line in a UTM zone. Bomford (1962) compared this formula with others over a known test line and recommended its use. For most practical purposes, the term in braces { } in equation (16) is omitted as its effect is negligible. For a line 100 km in length running north and south on a zone boundary the error in neglecting this term is about 0.08" (NMC 1985).

The arc-to-chord correction at P_2, for the line P_2 to P_1, is designated as δ_{21} and will be of opposite sign to δ_{12} and slightly different in magnitude. The arc-to-chord correction, grid bearing and plane bearing are related by

$$\theta = \beta + \delta \tag{17}$$

The grid convergence (given by Redfearn's equations) and the arc-to-chord corrections have a sign convention when used in Australia, given by the relationships in equations (14) and (17). Often the sign of these quantities can be ignored and the correct relationships determined from a simple diagram.

REFERENCES

Bomford, A.G., 1962. 'Transverse Mercator arc-to-chord and finite distance scale factor formulae', *Empire Survey Review,* No. 125, Vol. XVI, pp. 318-327, July 1962.

Featherstone, W.E. and Kirby, J.F., 2002. 'Short Note: Traverse computation on the ellipsoid instead of on the map plane', *The Australian Surveyor,* Vol. 47, No. 1, pp.38-42, June 2002.

ICSM, 2002. *Geocentric Datum of Australia Technical Manual,* Version 2.2, Intergovernmental Committee on Surveying & Mapping (ICSM), Canberra, February 2002.
http://www.icsm.gov.au/icsm/gda/gdatm/gdav2.2.pdf
(last accessed 15 July 2004)

NMC, 1985. *The Australian Geodetic Datum Technical Manual,* Special Publication 10, National Mapping Council of Australia, Canberra, 1985

NMC, 1972. *The Australian Map Grid Technical Manual,* Special Publication 7, National Mapping Council of Australia, Canberra, 1972

Redfearn, J.C.B., 1948. 'Transverse Mercator formula', *Empire Survey Review,* No. 69, Vol. IX, pp. 318-22, July 1948.

Thomas, P.D., 1952. *Conformal Projections in Geodesy and Cartography,* Special Publication No. 251, Coast and Geodetic Survey, United States Department of Commerce, Washington.

EXAMPLE TRAVERSE COMPUTATIONS ON THE UTM PROJECTION PLANE

Using the reduced traverse observations (geodesic distances and geodesic angles) and coordinates of the fixed stations *Smeaton, Buninyong, Arthur's Seat* and *Bass,* the MGA94 East and North coordinates of the traverse stations are computed using the Excel spreadsheet Redfearn.xls and equations (10) to (17) in a defined sequence. The method of computation is iterative and each leg of the traverse is computed before proceeding to the next leg. A diagram for each traverse leg is essential.

The reduced traverse observations are shown in Table 22

	Station / Geodesic Distance	Geodesic angle	
1	Smeaton		
2	Buninyong	1-2-3	119°47´10.06″
	(2-3) 54972.161 m		
3	Flinders Peak	2-3-4	196°43´49.44″
	(3-4) 27659.183 m		
4	Berllarine	3-4-5	163°45´32.33″
	(4-5) 37175.169 m		
5	Arthur's Seat	4-5-6	158°34´37.46″
6	Bass		

Table 22. Geodesic distances and angles of the *Buninyong-Arthur's Seat* traverse

The traverse diagram is shown in Figure 8 with MGA94 coordinates of the fixed stations in the traverse. It is required to compute the MGA94 coordinates of *Flinders Peak* and *Bellarine.*

LINE 1 *Buninyong-Flinders Peak*

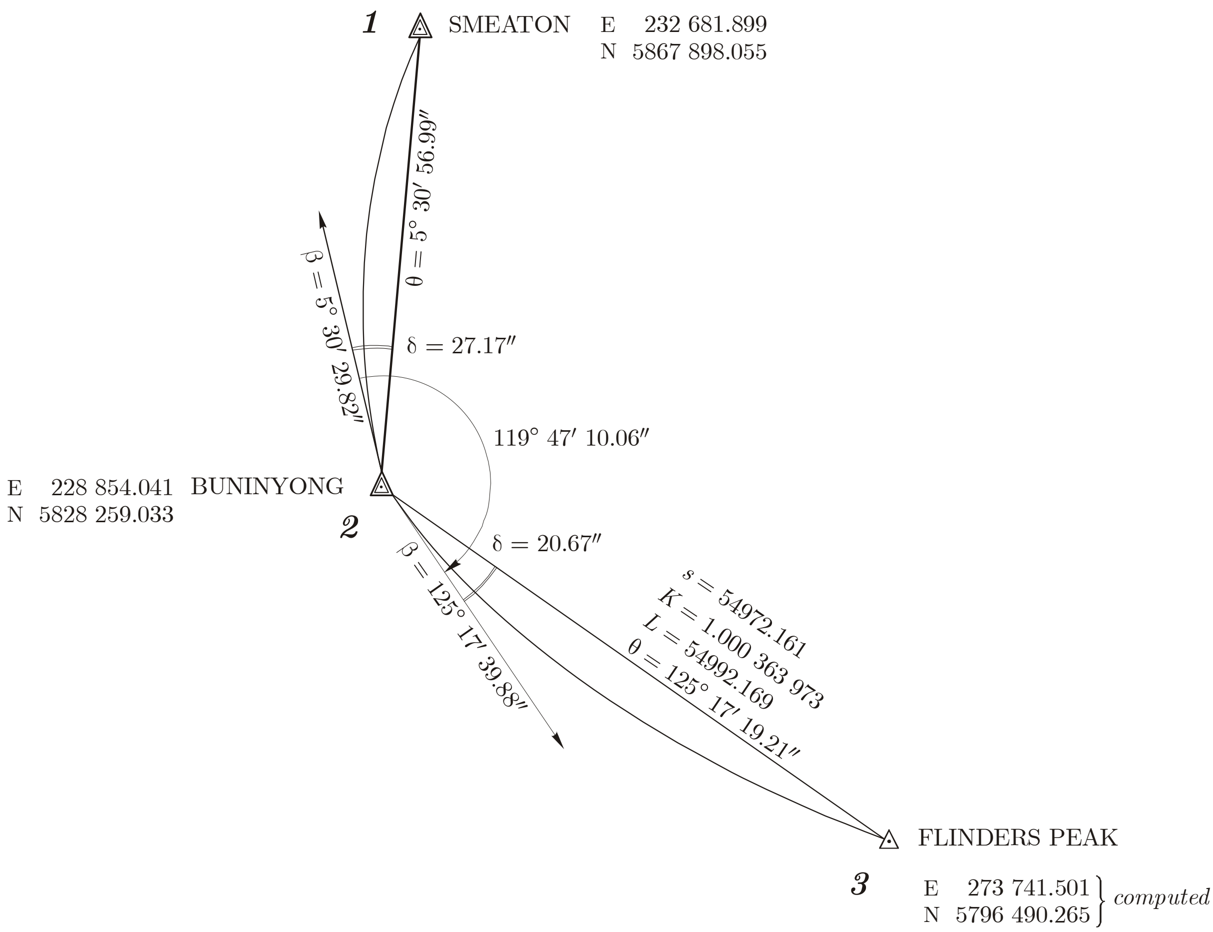

Figure 12 Traverse lines *Buninyong–Smeaton* and *Buninyong–Flinders Peak.*

Figure 12 shows the traverse lines *Buninyong–Smeaton* and *Buninyong–Flinders Peak* with computed values of relevant plane bearings (θ), arc-to-chord corrections (δ), grid bearings (β), geodesic distance (s), line scale factor (K), plane distance (L) and computed coordinates (E, N) of *Flinders Peak.* The quantities on the diagram are computed in the following sequence.

STEP 1 Compute the plane bearing of the back-sight line *Buninyong-Smeaton*

Using the MGA94 coordinates from the traverse diagram (Figure 8) and equation (15) the plane bearing θ of the line *Buninyong-Smeaton* is shown in Table 23

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	Plane Bearing 2-1
Buninyong	228854.041 E 5828259.033 N	Smeaton	232681.899 E 5867898.055 N	5°30´56.99˝

Table 23. Plane bearing of the line *Buninyong–Smeaton*.

STEP 2 Compute the arc-to-chord correction δ of the back-sight line *Smeaton–Buninyong.*

The arc-to-chord correction is computed using equation (16). This equation requires the mean radius $r_m = k_0\sqrt{\rho\nu}$ where ρ and ν are computed for $\phi_m = (\phi_1 + \phi_2)/2$.

2.1 Use the Excel spreadsheet Redfearn.xls to convert MGA94 *E,N* coordinates of the instrument station *Buninyong,* and back-sight station *Smeaton,* to GDA94 latitudes and longitudes. The grid convergence γ and the Point Scale Factor *k* should also be noted. The grid convergence is not used in these calculations but is computed from Redfearn's formulae. The Point Scale Factor *k* of the instrument station is used as a first approximation of the Line Scale Factor for the line *Buninyong–Flinders Peak.* The computed values are shown in Table 24.

Station	MGA94 coordinates	GDA94 coordinates	Grid convergence γ	Point Scale Factor k
Buninyong	228854.041 E 5828259.033 N	-37°39´10.1563˝ 143°55´35.3835˝	-1°52´43.22˝	1.000505669
Smeaton	232681.899 E 5867898.055 N	-37°17´49.7306˝ 143°59´03.1691˝		

Table 24. MGA Zone 55 and GDA coordinates, grid convergence γ and Point Scale Factor *k* of *Buninyong* and *Smeaton.*

2.2 Calculate the mean latitude r_m of the line *Buninyong–Smeaton* and use Redfearn.xls to compute ρ and ν. Then compute $r_m = k_0\sqrt{\rho\nu}$. Table 25 shows the computed values with r_m rounded to the nearest metre.

Line (1-2)	Mean latitude	Radii of curvature		Mean radius $r_m = k_0\sqrt{\rho\nu}$
		ρ	ν	
1 Buninyong- 2 Smeaton	-37°28´29.9434˝	6359061.793	6386054.391	6369995

Table 25. Mean radius r_m of the line *Buninyong–Smeaton.*

2.3 Calculate the arc-to-chord correction at *Buninyong* for the line *Buninyong–Smeaton* using equation (16). Table 26 shows the computed value

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord correction 1-2
Buninyong	228854.041 E 5828259.033 N	Smeaton	232681.899 E 5867898.055 N	6369995	27.17˝

Table 26. Arc-to-chord correction at *Buninyong* for the line *Buninyong–Smeaton.*

STEP 3 Compute the grid bearing β of the back-sight line *Buninyong–Smeaton.*

The grid bearing of the line *Buninyong–Smeaton* is given by equation (17) using values in Tables 23 and 26.

$$\beta = \theta - \delta = 5^\circ\, 30'\, 56.99'' - 27.17'' = 5^\circ\, 30'\, 29.82''$$

STEP 4 Compute the grid bearing β of the forward-sight line *Buninyong–Flinders Peak.*

The grid bearing of the forward-sight is equal to the grid bearing of the back-sight plus the geodesic angle at the instrument point from Table 22.

$$\beta = 5^\circ\, 30'\, 29.82'' + 119^\circ\, 47'\, 10.06'' = 125^\circ\, 17'\, 39.88''$$

STEP 5 Compute the MGA94 *E,N* coordinates of *Flinders Peak.*

This step requires an iterative approach.

The first iteration is made as follows:

(1a) The grid bearing of the line *Buninyong–Flinders Peak* is assumed to be the plane bearing θ.

(1b) The geodesic distance s of the line *Buninyong–Flinders Peak,* multiplied by the Point Scale Factor k at *Buninyong,* is assumed to be the plane distance L.

(1c) Using these approximations for θ and L and plane trigonometry, approximate coordinates of *Flinders Peak* are computed.

The second iteration is made as follows:

(2a) Convert the approximate grid coordinates of *Flinders Peak* to latitudes and longitudes using Redfearn.xls, calculate the mean latitude and the mean radius r_m for the line *Buninyong–Flinders Peak* as per steps 2.1 and 2.2 above.

(2b) Compute the arc-to-chord correction δ at *Buninyong* and the Line Scale Factor K for the line *Buninyong–Flinders Peak* using equations (16) and (13).

(2c) Compute new values for the plane bearing $\theta = \beta + \delta$ and plane distance $L = K \times s$ then use plane trigonometry to compute new approximations of the coordinates of *Flinders Peak.*

The third iteration is made as follows:

(3a) Compute the arc-to-chord correction δ at *Buninyong* and the Line Scale Factor L for the line *Buninyong–Flinders Peak* using equations (16) and (13). The mean radius r_m computed in the second iteration can be used since its value, rounded to the nearest metre, will not change. It is likely that the Line Scale Factor will remain unchanged.

(3b) Compute new values for the plane bearing $\theta = \beta + \delta$ and plane distance $L = K \times s$ then use plane trigonometry to compute new approximations of the coordinates of *Flinders Peak.*

The changes between the second and third iterations will generally be less than 1-2 mm. The results of the iterative process are set out below.

5.1 First Iteration

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Buninyong 228854.041 E 5828259.033 N	125°17´39.88˝	s = 54972.161 K = 1.000505669 L = 54999.959	Flinders Peak 273744.675 E 5796481.266 N

Table 27. First approximation of MGA94 coordinates of *Flinders Peak*.

5.2 Second Iteration

Station	MGA94 coordinates	GDA94 coordinates
Flinders Peak	273744.675 E 5796481.266 N	-37°57´03.9992˝ 144°25´29.6531˝

Table 28. Approximate MGA94 and GDA94 coordinates of *Flinders Peak*.

Line (1-2)	Mean latitude	Radii of curvature		Mean radius $r_m = k_0\sqrt{\rho\nu}$
		ρ	ν	
1 Buninyong 2 Flinders Peak	-37°48´07.0778˝	6359415.132	6386172.668	6370231

Table 29. Approximate mean radius r_m of the line *Buninyong–Flinders Peak*.

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord L.S.F
Buninyong	228854.041 E 5828259.033 N	Flinders Peak	273744.675 E 5796481.266 N	6370231	-20.68˝ 1.000363963

Table 30. First approximations of arc-to-chord correction and Line Scale Factor for the line *Buninyong–Flinders Peak*.

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Buninyong 228854.041 E 5828259.033 N	125°17´19.20˝	s = 54972.161 K = 1.000363963 L = 54992.169	Flinders Peak 273741.502 E 5796490.267 N

Table 31. Second approximation of MGA94 coordinates of *Flinders Peak*.

5.3 Third Iteration

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord L.S.F
Buninyong	228854.041 E 5828259.033 N	Flinders Peak	273741.502 E 5796490.267 N	6370231	-20.67″ 1.000363973

Table 32. Second approximations of arc-to-chord correction and Line Scale Factor for the line *Buninyong–Flinders Peak*.

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Buninyong 228854.041 E 5828259.033 N	125°17´19.21″	s = 54972.161 K = 1.000363973 L = 54992.169	Flinders Peak 273741.501 E 5796490.265 N

Table 33. Third approximation of MGA94 coordinates of *Flinders Peak*.

The MGA94 *E,N* coordinates of *Flinders Peak* from the third iteration (Table 33) can be regarded as "exact" since there has been only 1-2 mm changes between the second and third iterations. Figure 12 shows a schematic view of the traverse showing final quantities. Inspection of the results, after the iterative process, shows that there is mm agreement between this method, *Traverse Computation on the UTM Map Plane,* and the previously demonstrated method *Traverse Computation on the Ellipsoid* (see Table 21), but the computational workload is very much greater.

LINE 2 *Flinders Peak–Bellarine*

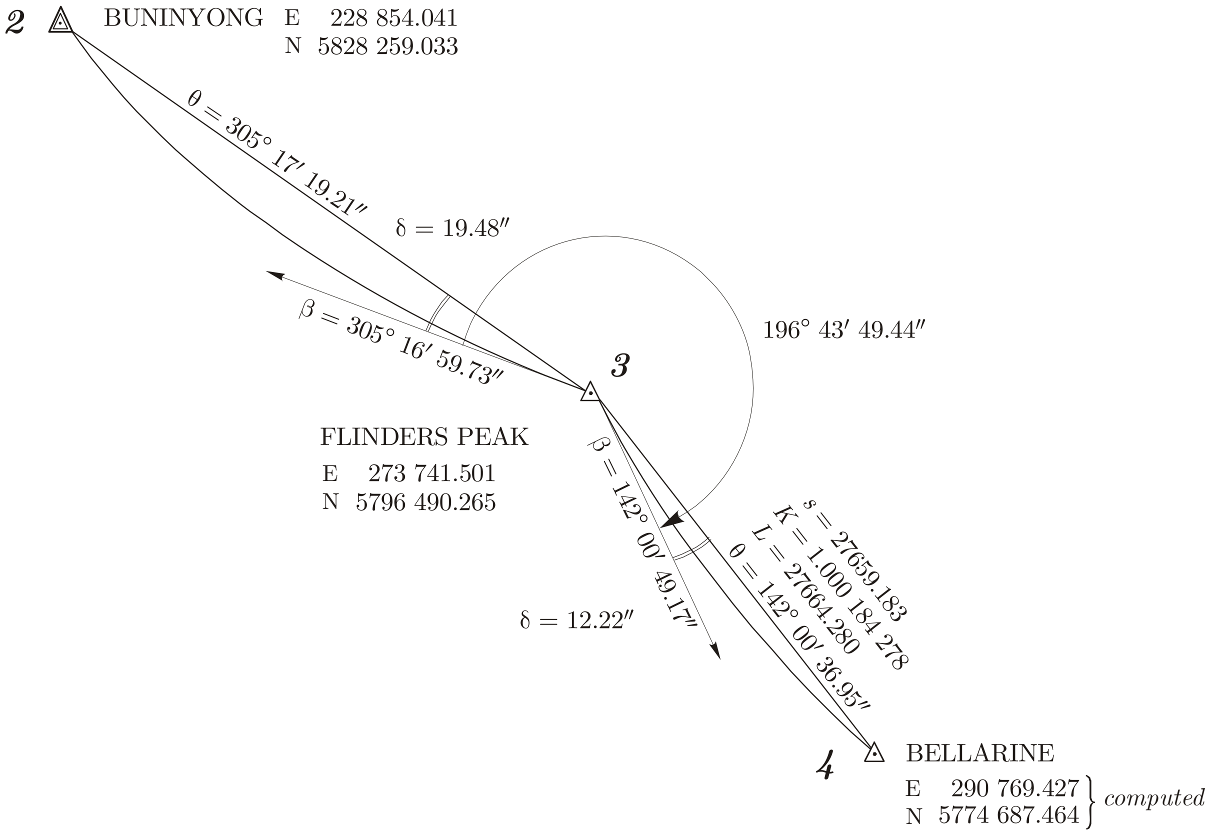

Figure 13 Traverse lines *Flinders Peak–Buninyong* and *Flinders Peak–Bellarine.*

Figure 13 shows the traverse lines *Flinders Peak–Buninyong* and *Flinders Peak–Bellarine* with computed values of relevant plane bearings (θ), arc-to-chord corrections (δ), grid bearings (β), geodesic distance (s), line scale factor (K), plane distance (L) and computed coordinates (E, N) of *Bellarine.* The quantities on the diagram are computed in the same sequence as set out for LINE 1 *Buninyong–Flinders Peak* and the abbreviated steps and results are as follows.

STEP 1 Compute the plane bearing of the back-sight line *Flinders Peak–Buninyong*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	Plane Bearing 1-2
Flinders Peak	273741.501 E 5796490.265 N	Buninyong	228854.041 E 5828259.033 N	305°17´19.21˝

Table 34. Plane bearing of the line *Flinders Peak–Buninyong.*

STEP 2 Compute the arc-to-chord correction δ of the back-sight line *Flinders Peak–Buninyong.*

2.1 Convert MGA94 *E,N* coordinates of the instrument station *Flinders Peak,* and back-sight station *Buninyong,* to GDA94 latitudes and longitudes. The Point Scale Factor k of the instrument station is used as a first approximation of the Line Scale Factor for the line *Flinders Peak–Bellarine.*

Station	MGA94 coordinates	GDA94 coordinates	Grid convergence γ	Point Scale Factor k
Flinders Peak	273741.501 E 5796490.265 N	-37°57´03.7047˝ 144°25´29.5333˝	-1°35´03.64˝	1.000230557
Buninyong	228854.041 E 5828259.033 N	-37°39´10.1563˝ 143°55´35.3835˝		

Table 35. MGA Zone 55 and GDA coordinates, grid convergence γ and Point Scale Factor k of *Flinders Peak.*

2.2 Calculate the mean latitude r_m of the line *Flinders Peak–Buninyong.*

Line (1-2)	Mean latitude	Radii of curvature		Mean radius
		ρ	ν	$r_m = k_0\sqrt{\rho\nu}$
1 Flinders Peak 2 Buninyong	-37°48´06.9305˝	6359415.088	6386172.654	6370231

Table 36. Mean radius r_m of the line *Flinders Peak–Buninyong.*

Alternatively, use the value of the mean radius r_m computed previously for the line *Buninyong–Flinders Peak* (see Table 36).

2.3 Calculate the arc-to-chord correction at *Flinders Peak* for the line *Flinders Peak–Buninyong.*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord correction 1-2
Flinders Peak	273741.501 E 5796490.265 N	Buninyong	228854.041 E 5828259.033 N	6370231	19.48″

Table 37. Arc-to-chord correction at *Flinders Peak* for the line *Flinders Peak–Buninyong.*

STEP 3 Compute the grid bearing β of the back-sight line *Flinders Peak–Buninyong.*

$$\beta = \theta - \delta = 305^\circ\, 17'\, 19.21'' - 19.48'' = 305^\circ\, 16'\, 59.73''$$

STEP 4 Compute the grid bearing β of the forward-sight line *Flinders Peak–Bellarine.*

$$\beta = 305^\circ\, 16'\, 59.73'' + 196^\circ\, 43'\, 49.44'' = 142^\circ\, 00'\, 49.17''$$

STEP 5 Compute the MGA94 *E,N* coordinates of *Bellarine.*

This step requires iteration.

5.1 First Iteration Compute approximate coordinates of *Bellarine* assuming the plane bearing θ is the grid bearing β and line scale factor K is the point scale factor k

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Flinders Peak 273741.501 E 5796490.265 N	142°00´49.17″	s = 27659.183 K = 1.000230557 L = 27665.560	Bellarine 290768.923 E 5774685.447 N

Table 38. First approximation of MGA94 coordinates of *Bellarine.*

5.2 Second Iteration

Use the first approximation of MGA94 coordinates of *Bellarine,* convert to GDA94 coordinates and compute r_m. Then compute arc-to-chord correction δ, Line Scale Factor K and "new" coordinates of *Bellarine*

Station	MGA94 coordinates	GDA94 coordinates
Bellarine	290768.923 E 5774685.447 N	-38°09´05.2655˝ 144°36´43.6716˝

Table 39. Approximate MGA94 and GDA94 coordinates of *Bellarine.*

Line (1-2)	Mean latitude	Radii of curvature		Mean radius
		ρ	ν	$r_m = k_0\sqrt{\rho\nu}$
1 Flinders Peak 2 Bellarine	-38°03´04.4851˝	6359685.227	6386263.077	6370411

Table 40. Approximate mean radius r_m of the line *Flinders Peak–Bellarine.*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord L.S.F
Flinders Peak	273741.501 E 5796490.265 N	Bellarine	290768.923 E 5774685.447 N	6370501	-12.22˝ 1.000184279

Table 41. First approximations of arc-to-chord correction and Line Scale Factor for the line *Flinders Peak–Bellarine.*

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Flinders Peak 273741.501 E 5796490.265 N	142°00´36.95˝	s = 27659.183 K = 1.000184279 L = 27664.280	Bellarine 273769.427 E 5774687.464 N

Table 42. Second approximation of MGA94 coordinates of *Bellarine.*

5.3 Third Iteration Repeat the procedure of the Second Iteration using the same value for r_m

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord / L.S.F
Flinders Peak	273741.501 E 5796490.265 N	Bellarine	290769.427 E 5774687.464 N	6370411	-12.22″ 1.000184278

Table 43. Second approximations of arc-to-chord correction and Line Scale Factor for the line *Flinders Peak–Bellarine*.

Station 1	Plane Bearing 3-4	Plane Distance $L = K \times s$	Station 2
Flinders Peak 273741.501 E 5796490.265 N	142°00´36.95″	s = 27659.183 K = 1.000184278 L = 27664.280	Bellarine 290769.427 E 5774687.464 N

Table 44. Third approximation of MGA94 coordinates of *Bellarine*.

The MGA94 *E,N* coordinates of *Bellarine* from the third iteration (Table 44) can be regarded as "exact" since there has been no changes between the second and third iterations. Figure 13 shows a schematic view of the traverse showing final quantities.
Inspection of the results, after the iterative process, shows that there is mm agreement between this method, *Traverse Computation on the UTM Map Plane,* and the previously demonstrated method *Traverse Computation on the Ellipsoid* (see Table 21).

LINE 3 *Bellarine–Arthur's Seat*

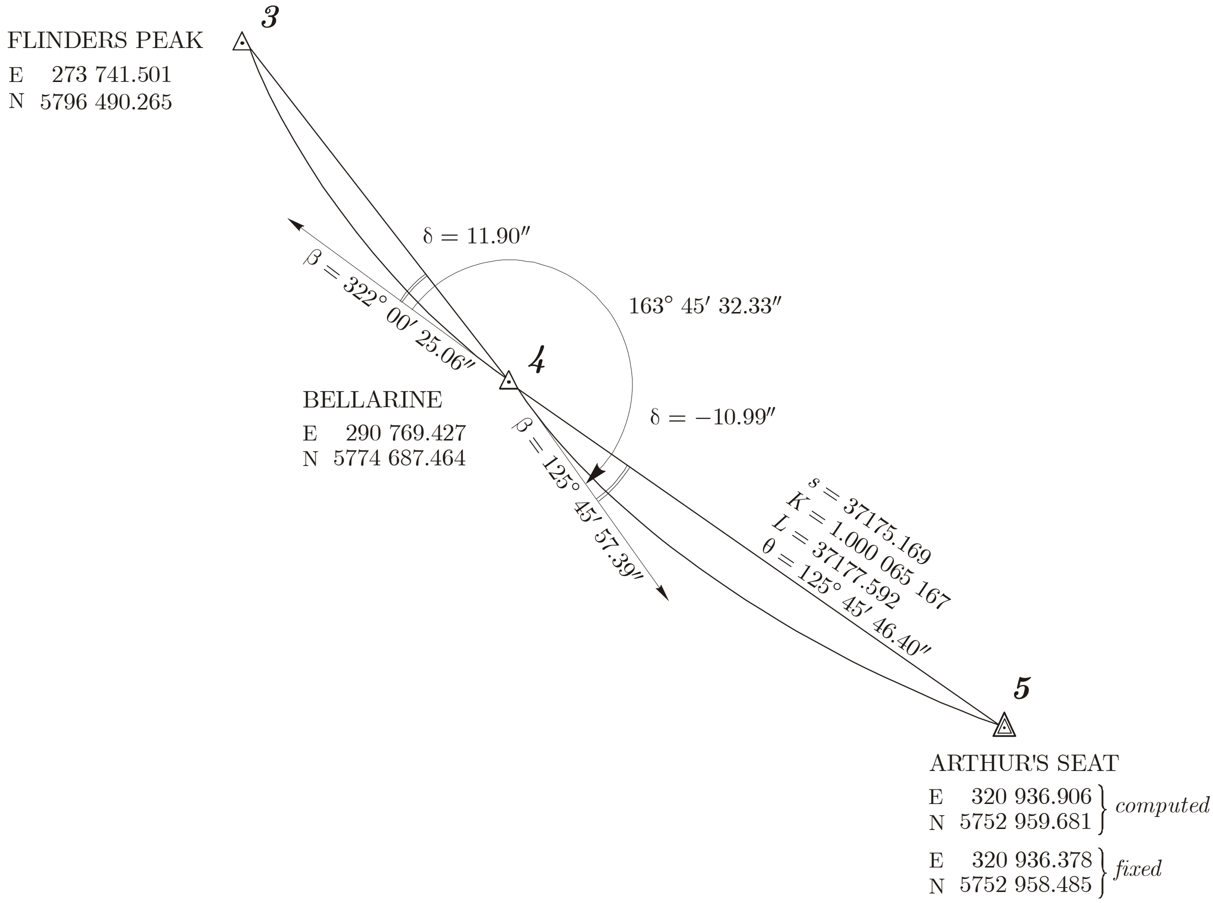

Figure 14 Traverse lines *Bellarine–Flinders Peak* and *Bellarine–Arthur's Seat.*

Figure 14 shows the traverse lines *Bellarine–Flinders* and *Bellarine–Arthur's Seat* with computed values of relevant plane bearings (θ), arc-to-chord corrections (δ), grid bearings (β), geodesic distance (s), line scale factor (K), plane distance (L) and computed coordinates (E, N) of *Arthur's Seat.* The quantities on the diagram are computed in the same sequence as set out for LINE 1 *Buninyong–Flinders Peak* and the abbreviated steps and results are as follows.

STEP 1 Compute the plane bearing of the back-sight line *Bellarine–Flinders Peak*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	Plane Bearing 1-2
Bellarine	290769.427 E 5774687.464 N	Flinders Peak	273741.501 E 5796490.265 N	322°00´36.95˝

Table 45. Plane bearing of the line *Bellarine–Flinders Peak.*

STEP 2 Compute the arc-to-chord correction δ of the back-sight line *Bellarine–Flinders Peak.*

2.1 Convert MGA94 *E,N* coordinates of the instrument station *Bellarine,* and back-sight station *Flinders Peak,* to GDA94 latitudes and longitudes. The Point Scale Factor k of the instrument station is used as a first approximation of the Line Scale Factor for the line *Bellarine–Arthur's Seat.*

Station	MGA94 coordinates	GDA94 coordinates	Grid convergence γ	Point Scale Factor k
Bellarine	290769.427 E 5774687.464 N	-38°09´05.2005˝ 144°36´43.6944˝	-1°28´32.23˝	1.000139186
Flinders Peak	273741.501 E 5796490.265 N	-37°57´03.7047˝ 144°25´29.5333˝		

Table 46. MGA Zone 55 and GDA coordinates, grid convergence γ and Point Scale Factor k of *Bellarine.*

2.2 Calculate the mean latitude r_m of the line *Bellarine–Flinders Peak.*

Line (1-2)	Mean latitude	Radii of curvature		Mean radius
		ρ	ν	$r_m = k_0\sqrt{\rho\nu}$
1 Bellarine 2 Flinders Peak	-38°03´04.4526˝	6359685.217	6386263.074	6370411

Table 47. Mean radius r_m of the line *Bellarine–Flinders Peak.*

Alternatively, use the value of the mean radius r_m computed previously for the line *Flinders Peak–Bellarine* (see Table 40).

2.3 Calculate the arc-to-chord correction at *Bellarine* for the line *Bellarine–Flinders Peak.*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord correction 1-2
Bellarine	290769.427 E 5774687.464 N	Flinders Peak	273741.501 E 5796490.265 N	6370411	11.90˝

Table 48. Arc-to-chord correction at *Bellarine* for the line *Bellarine–Flinders Peak.*

STEP 3 Compute the grid bearing β of the back-sight line *Bellarine–Flinders Peak.*

$$\beta = \theta - \delta = 322^\circ\, 00'\, 36.95'' - 11.90'' = 322^\circ\, 00'\, 25.06''$$

STEP 4 Compute the grid bearing β of the forward-sight line *Bellarine–Arthur's Seat.*

$$\beta = 322^\circ\, 00'\, 25.06'' + 163^\circ\, 45'\, 32.33'' = 125^\circ\, 45'\, 57.39''$$

STEP 5 Compute the MGA94 *E,N* coordinates of *Arthur's Seat.*

This step requires iteration.

5.1 First Iteration Compute approximate coordinates of *Arthur's Seat* assuming the plane bearing θ is the grid bearing β and line scale factor K is the point scale factor k

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Bellarine 290768.923 E 5774685.447 N	125°45´57.39˝	s = 37175.169 K = 1.000139186 L = 37180.343	Arthur's Seat 320937.981 E 5752956.466 N

Table 49. First approximation of MGA94 coordinates of *Arthur's Seat.*

5.2 Second Iteration Use the first approximation of MGA94 coordinates of *Arthur's Seat,* convert to GDA94 coordinates and compute r_m. Then compute arc-to-chord correction δ, Line Scale Factor K and "new" coordinates of *Arthur's Seat*

Station	MGA94 coordinates	GDA94 coordinates
Arthur's Seat	320937.981 E 5752956.466 N	-38°21´13.1930˝ 144°57´02.6191˝

Table 50. Approximate MGA94 and GDA94 coordinates of *Arthur's Seat.*

Line (1-2)	Mean latitude	Radii of curvature		Mean radius
		ρ	ν	$r_m = k_0\sqrt{\rho\nu}$
1 Bellarine 2 Arthur's Seat	-38°15´09.1968˝	6359903.785	6386336.234	6370557

Table 51. Approximate mean radius r_m of the line *Bellarine–Arthur's Seat.*

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord L.S.F
Bellarine	290769.427 E 5774687.464 N	Arthur's Seat	320937.981 E 5752956.466 N	6370557	-11.00˝ 1.000065164

Table 52. First approximations of arc-to-chord correction and Line Scale Factor for the line *Bellarine–Arthur's Seat.*

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Bellarine 290769.427 E 5774687.464 N	125°45´46.39˝	s = 37175.169 K = 1.000065164 L = 37177.591	Arthur's Seat 320936.907 E 5752959.684 N

Table 53. Second approximation of MGA94 coordinates of *Arthur's Seat.*

5.3 Third Iteration Repeat the procedure of the Second Iteration using the same value for r_m

Station 1	MGA94 coordinates	Station 2	MGA94 coordinates	r_m	arc-to-chord L.S.F
Bellarine	290769.427 E 5774687.464 N	Arthur's Seat	320936.907 E 5752959.684 N	6370557	-10.99″ 1.000065167

Table 54. Second approximations of arc-to-chord correction and Line Scale Factor for the line *Bellarine–Arthur's Seat*.

Station 1	Plane Bearing 1-2	Plane Distance $L = K \times s$	Station 2
Bellarine 290769.427 E 5774687.464 N	125°45´46.40″	s = 37175.169 K = 1.000065167 L = 37177.592	Arthur's Seat 320936.906 E 5752959.681 N

Table 55. Third approximation of MGA94 coordinates of *Arthur's Seat*.

The MGA94 *E,N* coordinates of *Arthur's Seat* from the third iteration (Table 55) can be regarded as "exact" since there has only mm changes between the second and third iterations. Figure 14 shows a schematic view of the traverse showing final quantities.
Inspection of the results, after the iterative process, shows that there is mm agreement between this method, *Traverse Computation on the UTM Map Plane*, and the previously demonstrated method *Traverse Computation on the Ellipsoid* (see Table 21).